Sheet (6)

Sampling Distribution (Sample mean, different between two mean, Sample proportion, different between two sample proportion)

1. Random samples of size 225 are drawn from a population with mean 100 and standard deviation 20. Find the mean and standard deviation of the sample mean.

Solution

$n = 225, \qquad \mu = 100, \qquad \sigma = 20.$

$$\mu_{\bar{X}} = \mu = 100, \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{225}} = \frac{4}{3} = 1.33.$$

2. A population has mean 75 and standard deviation 12.
   (a) Random samples of size 121 are taken. Find the mean and standard deviation of the sample mean.
   (b) How would the answers to part (a) change if the size of the samples were 400 instead of 121?

Solution

$\mu = 75, \qquad \sigma = 12.$

(a) $n = 121$

$$\mu_{\bar{X}} = \mu = 75, \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{121}} = 1.09.$$

(b) $n = 400$

$$\mu_{\bar{X}} = \mu = 75, \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{400}} = 0.6.$$

$\mu_{\bar{X}}$ stays the same, but $\sigma_{\bar{X}}$ decreases to 0.6.

3. A population has mean 128 and standard deviation 22. Find the probability that the mean of a sample of size 36 will be within 10 units of the population mean, that is, between 118 and 138.

Solution

$\mu = 128, \qquad \sigma = 22, \ \ n = 36.$

$$\mu_{\bar{X}} = \mu = 128, \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{22}{\sqrt{36}} = 3.67.$$

$$P(118 < \bar{X} < 138) = P\left(\frac{118-128}{3.67} < Z < \frac{138-128}{3.67}\right)$$
$$= P(-2.72 < Z < 2.72)$$
$$= P(Z < 2.72) - P(Z < -2.72) = 0.9967 - 0.0033$$
$$= 0.9934.$$

4. A population has mean 73.5 and standard deviation 2.5. Find the probability that the mean of a sample of size 30 will be less than 72.

Solution

$$\mu = 73.5, \qquad \sigma = 2.5, \;\; n = 30.$$

$$\mu_{\bar{X}} = \mu = 73.5, \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{2.5}{\sqrt{30}} = 0.456.$$

$$P(\bar{X} < 72) = P\left(Z < \frac{72-73.5}{0.456}\right) = P(Z < -3.29) = 0.0005.$$

5. A normally distributed population has mean 25.6 and standard deviation 3.3.
   (a) Find the probability that a single randomly selected element X of the population exceeds 30.
   (b) Find the probability that the mean of a sample of size 9 drawn from this population exceeds 30.

Solution

$$\mu = 25.6, \qquad \sigma = 3.3.$$

(a) $P(X > 30) = P\left(Z > \frac{30-25.6}{3.3}\right) = P(Z > 1.33) = 1 - P(Z < 1.33) = 1 - 0.9082 = 0.0918.$

(b) $n = 9$

$$\mu_{\bar{X}} = \mu = 25.6, \;\; \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{3.3}{\sqrt{9}} = 1.1.$$

$$P(\bar{X} > 30) = P\left(Z > \frac{30-25.6}{1.1}\right) = P(Z > 4) = 1 - P(Z < 4)$$
$$= 1 - 1 = 0.$$

6. Suppose the mean amount of cholesterol in eggs labeled “large” is 186 milligrams, with standard deviation 7 milligrams. Find the probability that the mean amount of cholesterol in a sample of 144 eggs will be within 2 milligrams of the population mean.

Solution

$\mu = 186, \quad \sigma = 7, \quad n = 144.$

$\mu_{\bar{X}} = \mu = 186, \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{7}{\sqrt{144}} = 0.583.$

$$P(184 < \bar{X} < 188) = P\left(\frac{184 - 186}{0.583} < Z < \frac{188 - 186}{0.583}\right)$$
$$= P(-3.43 < Z < 3.43)$$
$$= P(Z < 3.43) - P(Z < -3.43) = 0.9997 - 0.0003$$
$$= 0.9994.$$

7. The proportion of a population with a characteristic of interest is $p = 0.37$. Find the mean and standard deviation of the sample proportion $\hat{p}$ obtained from random samples of size 1600.

Solution

$p = 0.37, \quad q = 1 - p = 0.63, \quad n = 1600.$

$\mu_{\hat{p}} = p = 0.37, \quad \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.37)(0.63)}{1600}} = 0.0121.$

8. In one study it was found that 86% of all homes have a functional smoke detector. Suppose this proportion is valid for all homes. Find the probability that in a random sample of 600 homes, between 80% and 90% will have a functional smoke detector. You may assume that the normal distribution applies.

Solution

$p = 0.86, \quad q = 1 - p = 0.14, \quad n = 600.$

$\mu_{\hat{p}} = p = 0.86, \quad \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.86)(0.14)}{600}} = 0.0142.$

$$P(0.8 < \hat{p} < 0.9) = P\left(\frac{0.8 - 0.86}{0.0142} < Z < \frac{0.9 - 0.86}{0.0142}\right)$$
$$= P(-4.22 < Z < 2.82)$$
$$= P(Z < 2.82) - P(Z < -4.22) = 0.9976.$$

9. For boys, the average number of absences in the first grade is 15 with a standard deviation of 7; for girls, the average number of absences is 10 with a standard deviation of 6. In a nationwide survey, suppose 100 boys and 50 girls are sampled. What is the probability that the male

sample will have *at most* three more days of absences than the female sample?

Solution

$\mu_B = 15, \sigma_B = 7, \quad \mu_G = 10, \sigma_G = 6, \quad n_B = 100, n_G = 50.$

$\mu_{\bar{X}_B - \bar{X}_G} = \mu_{\bar{X}_B} - \mu_{\bar{X}_G} = \mu_B - \mu_G = 15 - 10 = 5.$

$$\sigma_{\bar{X}_B - \bar{X}_G} = \sqrt{\frac{\sigma_B^2}{n_B} + \frac{\sigma_G^2}{n_G}} = \sqrt{\frac{7^2}{100} + \frac{6^2}{50}} = \frac{11}{10} = 1.1$$

$$P(\bar{X}_B - \bar{X}_G \le 3) = P\left(Z \le \frac{3-5}{1.1}\right) = P(Z \le -1.82) = 0.0344.$$

10. In a certain area of a large city it is hypothesized that 40 percent of the houses are in a dilapidated condition. A random sample of 75 houses from this section and 90 houses from another section yielded a difference, $\hat{p}_1 - \hat{p}_2$, of .09. If there is no difference between the two areas in the proportion of dilapidated houses, what is the probability of observing a difference this large or larger?

Solution

$n_1 = 75, n_2 = 90, \quad \hat{p}_1 - \hat{p}_2 = 0.9, p_1 - p_2 = 0, \quad p_1 = p_2 = 0.4,$
$q_1 = q_2 = 0.6.$

$$P(\hat{p}_1 - \hat{p}_2 > 0.9) = P\left(Z \ge \frac{0.9 - 0}{\sqrt{\frac{(0.4)(0.6)}{75} + \frac{(0.4)(0.6)}{90}}}\right)$$
$$= P(Z \ge 1.17) = 1 - P(Z < 1.17) = 1 - 0.8790$$
$$= 0.121.$$