Sec “ 6 ”

Normal distribution

Binomial distribution

Random Variable:

Is a set of **possible values** from a random experiment.

Discrete Random Variable

- Is one which may take on only a countable number of distinct values such as 0,1,2,3,4,.........
- Example: The number of children in a family.

  $X = 0, 1, 2.$

Continuous Random Variable

- Is one which takes an infinite number of possible values.
- Example: Height.

  $150 \leq X \leq 180$

  Temperature.

  $40 \leq X \leq 50$

Example:

X: number of heads when the coin is tossed twice

$$X = \begin{cases} 0 \leftarrow \\ 1 \leftarrow \\ 2 \leftarrow \end{cases}$$

T T

H T, TH

H H

Probability distribution $[f(x)]$:

Is the mathematical function that gives the probabilities of occurrence of different possible outcomes for an experiment.

Probability Mass Function "$p.m.f$"

- Is a function that gives the probability that a discrete random variable is exactly equal to some value.
- $f(x)$ is a probability mass function if :

$$(a)\ 0 \le f(x) \le 1$$

$$(b)\ \sum_{x} f(x) = 1$$

- $P(X = a) = f(a).$

Probability Density Function "$p.d.f$"

- is used to define the probability of the random variable coming within a distinct range of values.
- $f(x)$ is a probability density function if :

$$(a)\ 0 \le \int_A f(x)\ dx \le 1 \quad , A \epsilon R$$

$$(b) \int_R f(x)\ dx = 1$$

- $P(a \le X \le b) = \int_a^b f(x)\ dx.$

Note That :

- For a discrete random variable $P(X = a) = 0$, if a is'nt a value in the random variable X.

  i.e., $\boldsymbol{X = 1, 3, 6}$

  any values except 1, 3, and 6 has a probability is equal to zero.

  $i.e., P(X = 2) = 0$.

- For a continuous random variable, $P(X = a) = 0$, for all $a \epsilon R$.

  i.e., $\boldsymbol{1 \le X \le 4}$

  $P(X = 2) = \int_2^2 f(x)\, dx = 0$.

and $P(a \le X \le b) = 0$, for all $a \le x \le b$ $not\ in$ R.

i.e., $P(1 \le X \le 6) = \int_1^4 f(x)\, dx + \int_4^6 \underset{0}{\cancel{f(x)}} dx = \int_1^4 f(x)\, dx$.

Example (1) :

x	0	1	3
$f(x)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

(a) Verify that $f(x)$ is a valid $p.m.f$?

$f(x)$ is a valid $p.m.f$ if $\sum_x f(x) = 1$. Thus,

$$f(0) + f(1) + f(3) = \frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1$$

It is a valid $p.m.f$.

(b) Find $P(1 < x \leq 3)$?

$$P(1 < x \leq 3) = P(X = 2) + P(X = 3) = 0 + \frac{1}{6} = \frac{1}{6}$$

Example (2) :

$$f(x)=\begin{cases}\frac{1}{3}x^2 & -1<x<2\\ 0 & otherwise\end{cases}.$$

(a) Verify that $f(x)$ is a valid $p.d.f$?

$f(x)$ is a valid $p.d.f$ if $\int_R f(x)=1$. Thus,

$$\int_{-1}^{2}\frac{1}{3}x^2=\left[\frac{1}{3}*\frac{x^3}{3}\right]_{-1}^{2}=\left[\frac{x^3}{9}\right]_{-1}^{2}=\frac{1}{9}[x^3]_{-1}^{2}=\frac{1}{9}[(2)^3-(-1)^3]=\frac{1}{9}[8-(-1)]=\frac{1}{9}(9)=1$$

It is a valid $p.d.f$.

(b) Find $P(0<x\leq 1)$?

$$P(0<x\leq 1)=\int_{0}^{1}\frac{1}{3}x^2=\frac{1}{9}[x^3]_0^1=\frac{1}{9}[(1)^3-(0)^3]=\frac{1}{9}$$

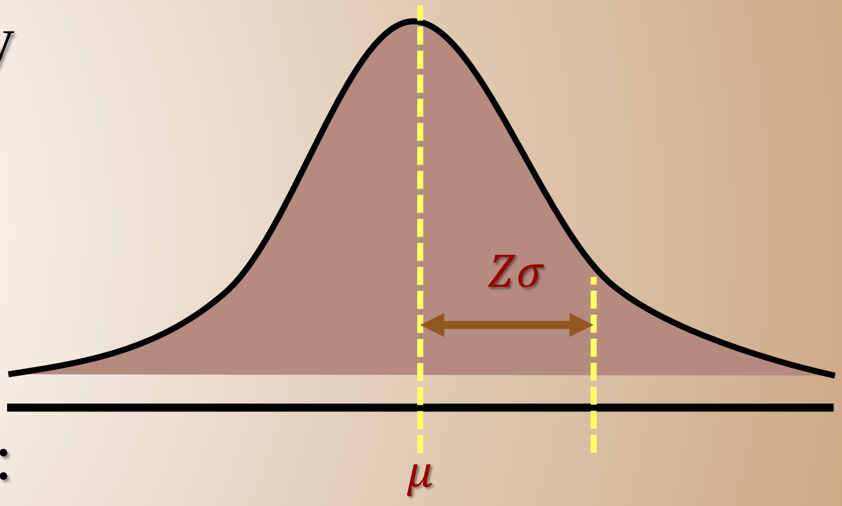
Normal distribution

- The data tends to be around a central value with no bias left or right, and it gets close to a "Normal Distribution" like this:

- x is a normal distribution, with probability density function is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty \le x \le \infty$$

$E(x) = \mu, \quad var(x) = \sigma^2,$ Standard Deviation of $x = \sigma$

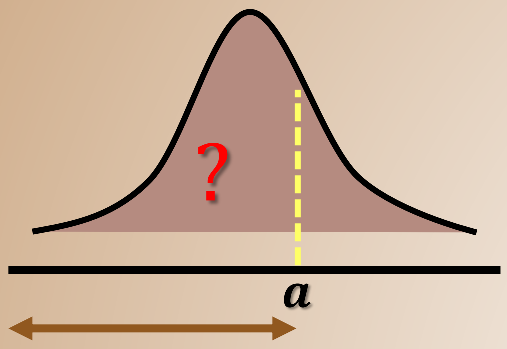


- Convert x " normal distribution " to Standard normal "Z":

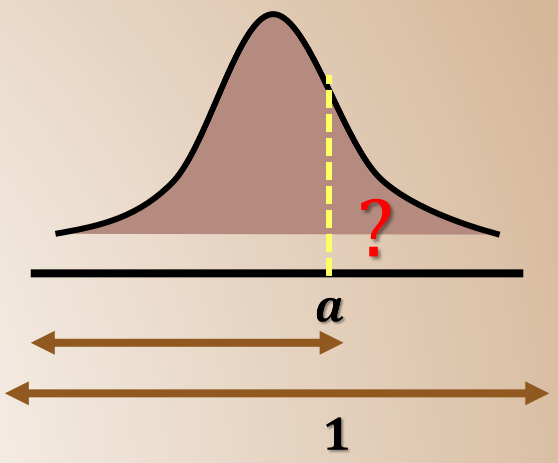
$$Z = \frac{x-\mu}{\sigma}$$

is a normal distribution with $\mu = 0, \quad \sigma^2 = 1$.

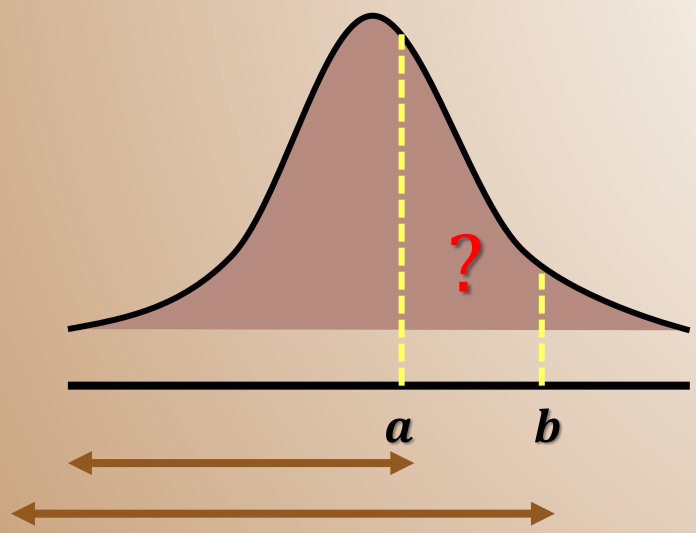
- $P(Z < a) = Z - Table$



- $P(Z > a) = 1 - P(Z < a)$



- $P(a < Z < b) = P(Z < b) - P(Z < a)$



- $P(Z < a) = 1 - P(Z < -a)$

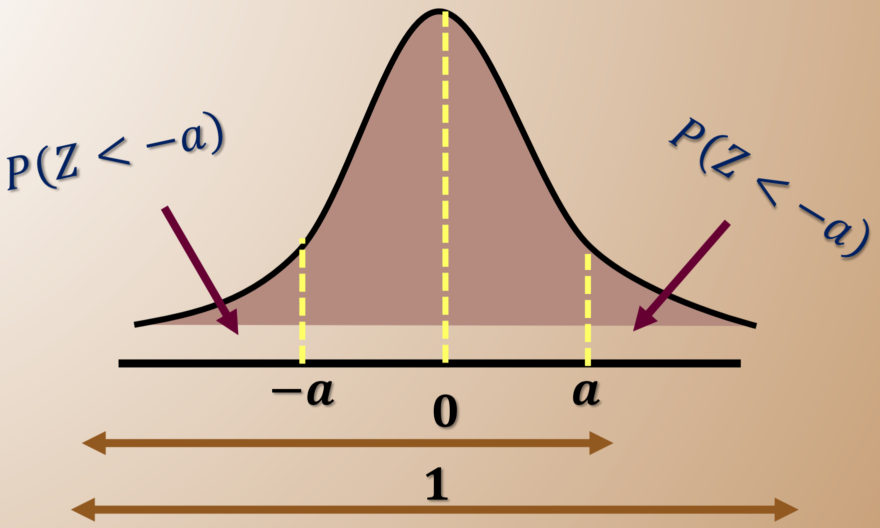


Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Note that:

- $P(Z < a) = 1 \longrightarrow$ if $a > 3.49$

  i.e., $P(Z < 5) = 1$.

- $P(Z < a) = 0 \longrightarrow$ if $a < -3.4$

  i.e., $P(Z < -5) = 0$.

Sheet (5)

12. A research scientist reports that mice will live **an average of 40 months** when their diets are sharply restricted and then enriched with vitamins and proteins. Assuming that the lifetimes of such mice are **normally distributed** with a **standard deviation of 6.3** months, find the probability that a given mouse will live

(a) More than 32 months;

(b) Less than 28 months;

(c) Between 37 and 49 months.

Solution

$\mu = 40, \qquad \sigma = 6.3$

$$\text{(a) } P(x > 32) = P\left(\frac{x-\mu}{\sigma} > \frac{32-40}{6.3}\right)$$
$$= P(Z > -1.2698)$$
$$= P(Z > -1.27)$$
$$= 1 - P(Z < -1.27)$$
$$= 1 - 0.1020 = 0.898$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379

Solution

$\mu = 40, \qquad \sigma = 6.3$

OR

(a) $P(x > 32) = 1 - P(Z < -1.27)$

$= 1 - \left(1 - P(Z < 1.27)\right)$

$= P(Z < 1.27)$

$= 0.898$

z	.00	.01	.02	.03	.04	.05	.06	$\boxed{.07}$	.08	.09
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
$\boxed{1.2}$	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	$\boxed{0.8980}$	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319

(b) $P(x < 28) = P\left(\frac{x-\mu}{\sigma} < \frac{28-40}{6.3}\right)$

$= P(Z < -1.904)$

$= P(Z < -1.90)$

$= 0.0287$

z	$\boxed{.00}$	.01	.02	.03	.04	.05	.06	.07	.08	.09
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
$\boxed{-1.9}$	$\boxed{0.0287}$	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.03.59	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367

Solution

$\mu = 40, \qquad \sigma = 6.3$

$$\text{(c) } P(37 < x < 49) = P\left(\frac{37-40}{6.3} < \frac{x-\mu}{\sigma} < \frac{49-40}{6.3}\right)$$
$$= P(-0.075 < Z < 0.226)$$
$$= P(-0.08 < Z < 0.23)$$
$$= P(Z < 0.23) - P(Z < -0.08)$$
$$= 0.5910 - 0.4681$$
$$= 0.1229$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	[illegible]	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5138	0.5478	[illegible]	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	$\boxed{0.5910}$	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	[illegible]	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	[illegible]	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	[illegible]	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	[illegible]	$\boxed{0.4681}$	0.4641

Sheet (5)

9. Wages for workers in a particular industry have an **average of \$11.90** per hour with a **standard deviation of 40 cents (1\$ = 100 cents)**. **The wages are considered to be normally distributed**.

(a) Suppose you are employed in this industry. What would your wage have to be if 75% of all workers earn more than you?

(b) What fraction of workers make between \$12 and \$13 per hour?

Solution

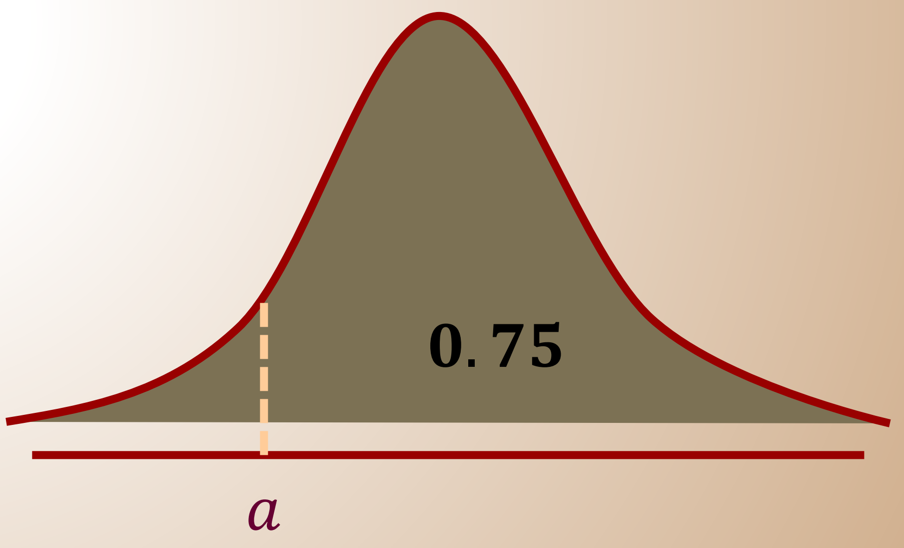
$\mu = \$11.90, \qquad \sigma = 40 \text{ cent} = \frac{40}{100} = \$\,0.4$

(a) $P(Z > a) = 0.75$

$\therefore \quad P(Z < a) = 1 - 0.75$

$= 0.25$

$a = ? \rightarrow P(Z < a) = 0.25$



Solution

$$\boldsymbol{P(Z < -0.67) = 0.25}$$

$$\therefore Z = -0.67$$

$$Z = \frac{x - \mu}{\sigma} \quad \rightarrow x = Z\sigma + \mu$$

$$\therefore x = (-0.67)(0.4) + 11.90$$

$$= 11.632$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776

$$0.2514 - 0.25 = \boxed{0.0014}$$

$$0.25 - 0.2483 = 0.0017$$

(b) $P(12 < x < 13) = P\left(\frac{12-11.9}{0.4} < \frac{x-\mu}{\sigma} < \frac{13-11.9}{0.4}\right)$

$= P(0.25 < Z < 2.75)$

$= P(Z < 2.75) - P(Z < 0.25)$

$= 0.9970 - 0.5987$

$= 0.3983$

Binomial distribution

- To study the number of successes in a sequence of n independent experiments.

- n: Total number of trails.
  p: Probability of success.
  q: Probability of failure.

$$p + q = 1$$

- x is a binomial distribution, with probability mass function is given by:

$$f(x) = P(x = r) = \binom{n}{r} p^r q^{n-r} \qquad r = 0, 1, \ldots, n$$

$E(x) = np, \quad var(x) = npq$

- i.e., Let

X: Number of tails

n: 10, $\quad p = 0.5$

$$P(X = 4)$$
$$= \binom{10}{4} (0.5)^4 (0.5)^{10-4}$$

- *expected number of tails*
  $= E(x) = np = 10(0.5)$
  $= 5$
- $Var\ (x) = npq$
  $= 10(0.5)(0.5) = 2.5$

Sheet (5)

1. In a certain city district **the need for money to buy drugs is stated as the reason for 75% of all thefts**. Find the probability that among the **next 5** theft cases reported in this district,
   (a) exactly 2 resulted from the need for money to buy drugs;
   (b) at most 3 resulted from the need for money to buy drugs.

Solution

$$\boldsymbol{n = 5, p = 0.75, q = 1 - 0.75 = 0.25}$$

(a) $P(x = 2) = \binom{5}{2}(0.75)^2(0.25)^{5-2}$

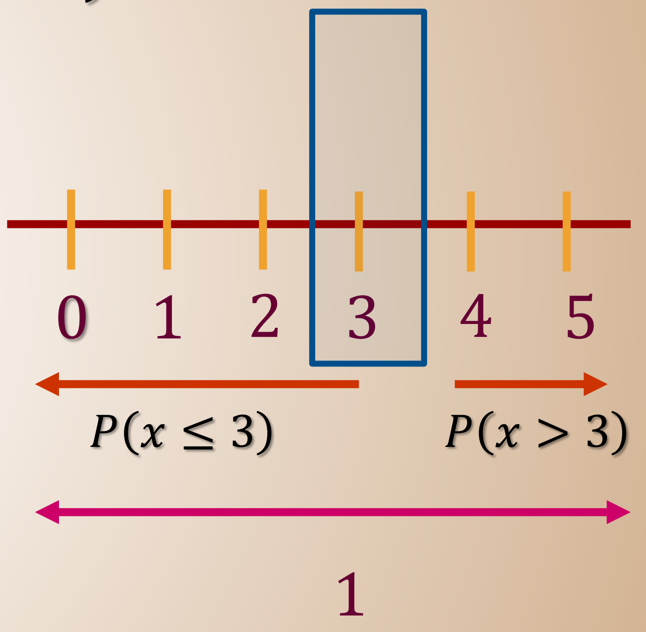
$$= \binom{5}{2}(0.75)^2(0.25)^3$$

$$= 0.087891$$

Solution

$\boldsymbol{n = 5, p = 0.75, q = 1 - 0.75 = 0.25}$

$$\begin{aligned}
\text{(b) } P(x \le 3) &= P(x=0) + P(x=1) + P(x=2) + P(x=3) \\
&= 1 - P(x > 3) \\
&= 1 - [P(x=4) + P(x=5)] \\
&= 1 - \left[\binom{5}{4}(0.75)^4(0.25)^1 + \binom{5}{5}(0.75)^5(0.25)^0\right] \\
&= 1 - [0.39551 + 0.23730] \\
&= 1 - 0.63281 = 0.36719
\end{aligned}$$



Sheet (5)

5. Suppose that airplane engines operate independently and **fail with probability equal to 0.4**. Assuming that **a plane makes a safe flight if at least one-half of its engines run**, determine whether a 4-engine plane or a 2-engine plane has the higher probability-for a successful flight.

Solution

$\boldsymbol{q = 0.4,\ p = 1 - 0.4 = 0.6}$

4-engine "$n = 4$"

$P(x \geq 2) = P(x = 2) + P(x = 3) + P(x = 4)$

$= 1 - P(x < 2) = 1 - [P(x = 0) + P(x = 1)]$

$= 1 - \left[\binom{4}{0}(0.6)^0(0.4)^4 + \binom{4}{1}(0.6)^1(0.4)^3\right]$

$= 1 - [0.0256 + 0.1536] = 1 - 0.1792$

$= 0.8208$

2-engine "$n = 2$"

$P(x \geq 1)$

$= P(x = 1) + P(x = 2)$

$= 1 - P(x < 1) = 1 - P(x = 0)$

$= 1 - \binom{2}{0}(0.6)^0(0.4)^2$

$= 1 - 0.16 = 0.84$

2-engine plane has the higher probability for successful flight.