SEC " 10 "

Testing a Single Mean

Statistical Hypothesis

- Hypothesis testing in statistics is a way for you to test the results of a survey or experiment to see if you have meaningful results. You’re basically testing whether your results are valid by figuring out the odds that your results have happened by chance. If your results may have happened by chance, the experiment won’t be repeatable and so has little use.

Hypothesis

- **<u>There are two hypotheses:</u>**

I. Null Hypothesis " H_0 " : Censoring a population parameter will always be stated so as to specify an exact value of the parameter.

H_0: population parameter $=$ given value " tested value ".

II. Alternative Hypothesis "H_1 ": Allows for the possibility of several values may be greater than, less than, or not equal.

H_1: population parameter $>$ given value " tested value ".

or

H_1: population parameter $<$ given value " tested value ".

or

H_1: population parameter $\neq$ given value " tested value ".

- In testing any statistical hypothesis, there are four possible situations that determine whether our decision is correct or in error. These four situations are summarized in the following table:

	H_0 true	H_0 false
Reject H_0	**Type I error**	**Correct decision**
Accept H_0	**Correct decision**	**Type II error**

Thus, the two types of error can happen in hypothesis-testing problems:

I. **Type I error:** Is rejection H_0 when H_0 it is actually true.

II. **Type II error:** Is accepting H_0 when H_0 it is actually false.

Hypothesis

The 7 Step Process of Statistical Hypothesis Testing:

1. State the Null Hypothesis that $H_0: \theta = \theta_0$.
2. Choose an appropriate alternative hypothesis from one of the alternatives

$$\left.\begin{array}{r} H_1: \theta > \theta_0, \\ H_1: \theta < \theta_0, \text{ or} \end{array}\right\} \quad \text{One tailed “one-sided” test}$$

$$H_1: \theta \neq \theta_0. \longrightarrow \text{Two tailed “two-sided” test}$$

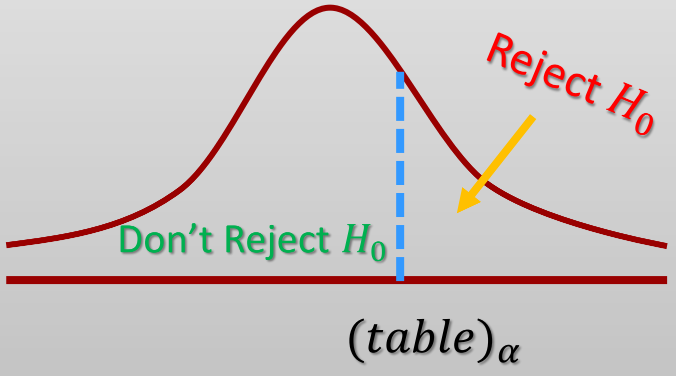
3. Choose a level of significance level of size α: The probability of committing a type I error “ the size of the critical region”.

α given (0.05 or 0.01), if not given set $\alpha = 0.05$

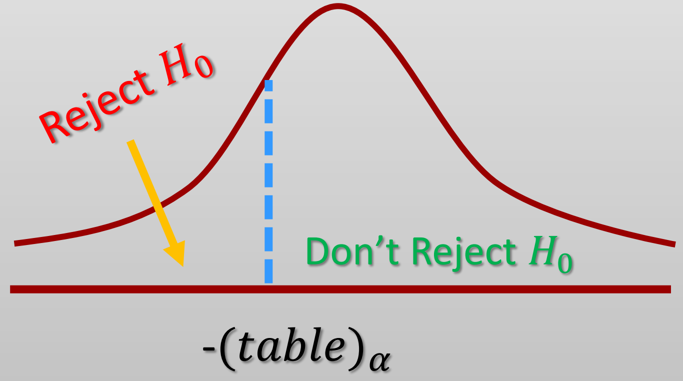
4. Compute the critical value “ the value that separate the region of rejection (critical region) from the remaining values ”: it is calculated from the z-table or t-table.

5. Critical Region:

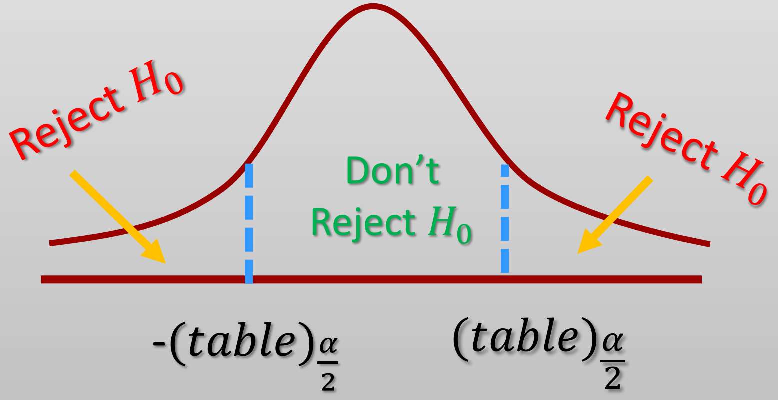
$H_1: \theta > \theta_0$, Right-tail test



$H_1: \theta < \theta_0$, Left−tail test



$H_1: \theta \neq \theta_0$, two−tailed test



6. Compute the test statistics from the sample data: a quantity, calculated from the sample information used as a basis for deciding whether or not to reject the null hypothesis.

7. Decision: Reject $\boldsymbol{H_0}$ if the test statistics has a value in the critical region; otherwise, do not reject $\boldsymbol{H_0}$.

Tests Concerning a Single Mean

Testing the hypothesis that the mean μ of a population, equals a specific value μ_0.

1. H_0: $\mu = \mu_0$.
2. H_1: $\mu > \mu_0$, H_1: $\mu < \mu_0$, or H_1: $\mu \neq \mu_0$. (if not given set H_1: $\mu \neq \mu_0$)
3. $\alpha = 0.05$ *or* 0.01. (if not given set $\alpha = 0.05$)
4. Compute the critical value from z-table or t-table, according to the following three cases:

Population Standard Deviation "$\boldsymbol{\sigma}$"

If $\boldsymbol{\sigma}$ known
Use: Z- table.

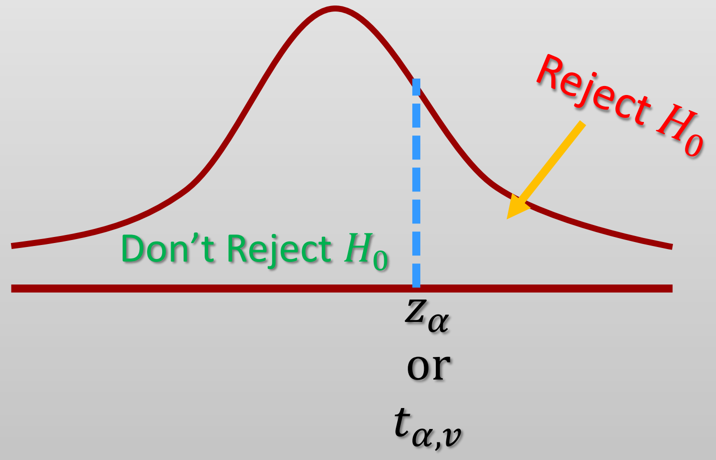
If $\boldsymbol{\sigma}$ unknown→ $\boldsymbol{s}$ known

If $n \geq 30$
Use: Z- table.

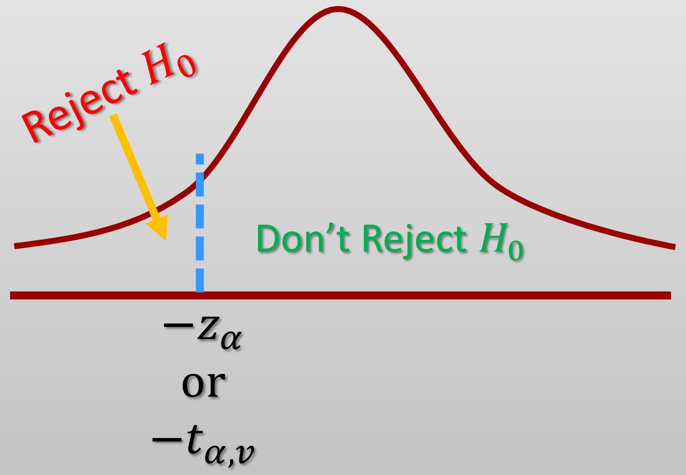
If $n < 30$
Use: **t-** table.

5. Critical Region:

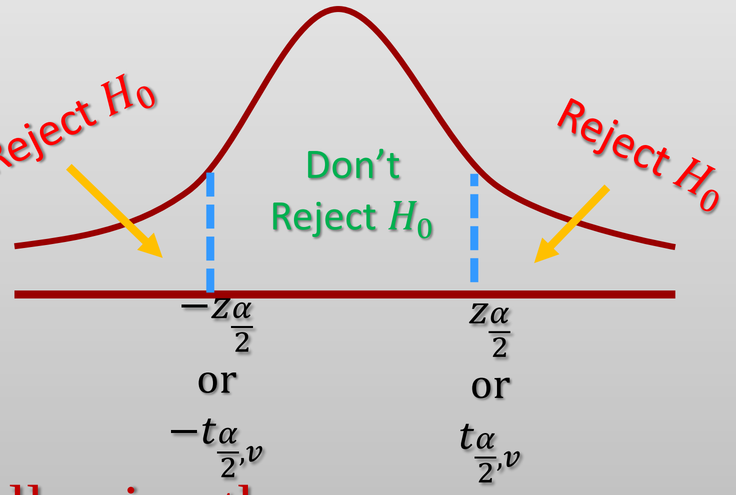
$H_1: \mu > \mu_0$, Right-tail test



$H_1: \mu < \mu_0$, Left−tail test



$H_1: \mu \neq \mu_0$, two−tailed test



6. Compute the test statistic value (Z or T), according to the following three cases:

Population Standard Deviation "σ"

If σ known

Compute: $Z = \dfrac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$

If σ unknown → s known

If $n \geq 30$

Compute: $Z = \dfrac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$

If $n < 30$

Compute: $T = \dfrac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$

7. Decision: compare the calculated statistic (Z or T) with the corresponding critical region limits such that:

Reject $\boldsymbol{H_0}$: if the calculated statistic (Z or T) falls in the rejection region.

not reject $\boldsymbol{H_0}$: otherwise.

Sheet (7)

2. An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. **Test the hypothesis that $\mu = 800$ hours against the alternative $\mu \neq 800$ hours** if a random sample of 30 bulbs has an average life of 788 hours.

Solution

$$\mu_0 = 800, \qquad \sigma = 40, \qquad n = 30, \qquad \bar{x} = 788$$

1. $H_0: \mu = 800.$
2. $H_1: \mu \neq 800.$
3. $\alpha = 0.05.$
4. Critical value:

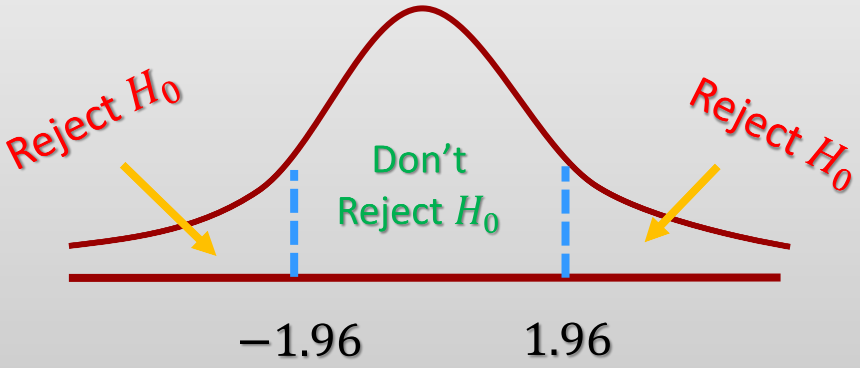
σ known → use z-table at $\frac{\alpha}{2}$ (two-tailed test)

$$z_{\frac{\alpha}{2}} = z_{\frac{0.05}{2}} = z_{0.025}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244

$$\boxed{\therefore z_{\frac{\alpha}{2}} = 1.96}$$

5. Critical Region: $H_1: \mu \neq 800$, two−tailed test



6. Compute the test statistic value (Z):

$\boldsymbol{\sigma}$ known, thus we compute:

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{788 - 800}{\frac{40}{\sqrt{30}}} = -1.64$$

7. Decision:

Since Z $= -1.64$ falls in the acceptance region, we not reject H_0 and conclude that the average life of bulbs is 800 ($\mu = 800$).

Sheet (7)

1. A random sample of 64 bags of white Cheddar popcorn weighed, on average, 5.23 ounces with a standard deviation of 0.24 ounces. **Test the hypothesis that $\mu = 5.5$ ounces against the alternative hypothesis, $\mu < 5.5$** ounces **at the 0.05 level of significance.**

Solution

$$\mu_0 = 5.5, \qquad n = 64, \qquad \bar{x} = 5.23, \qquad s = 0.24$$

1. H_0: $\mu = 5.5$.
2. H_1: $\mu < 5.5$.
3. $\alpha = 0.05$.
4. Critical value:

σ unknown and n $= 64 > 30 \rightarrow$
use z-table at α(One-tailed test)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.03.59	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0591	0.0582	0.0571	0.0559

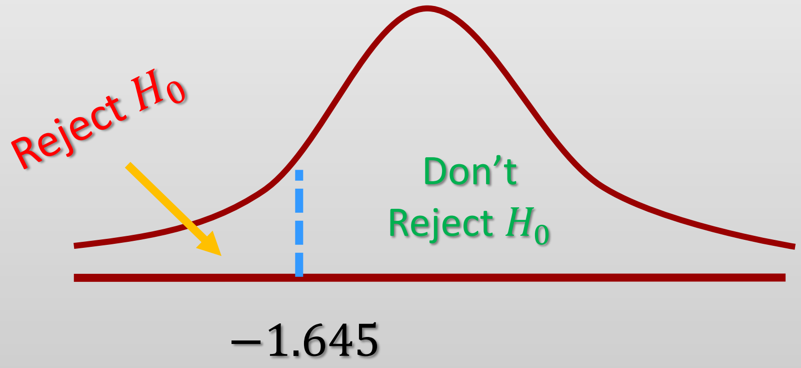
$$Z_\alpha = z_{0.05} = \frac{1.64 + 1.65}{2}$$

$$\boxed{\therefore \boldsymbol{z_\alpha = 1.645}}$$

$0.0505 - 0.05 = 0.0005$ ✓✓

$0.05 - 0.0495 = 0.0005$ ✓✓

5. Critical Region: $H_1: \mu < 5.5$, Left−tailed test



6. Compute the test statistic value (Z):

σ unknown and $n > 30$, thus we compute:

$$\boldsymbol{Z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{5.23 - 5.5}{\frac{0.24}{\sqrt{64}}} = -9}$$

7. Decision:

Since Z = −9 falls in the rejection region, we reject H_0 and conclude that the average weight of white Cheddar popcorn is less than 5.5 ($\mu < 5.5$).

Sheet (7)

5. It is claimed that an automobile is driven on the average more than 20,000 kilometers per year. To test this claim, a random sample of 100 automobile owners are asked to keep a record of the kilometers they travel. Would you agree with this claim if the random sample showed an average of 23,500 kilometers and a standard deviation of 3900 kilometers?

Solution

$$\mu_0 = 20000, \qquad n = 100, \qquad \overline{x} = 23500, \qquad s = 3900$$

1. $H_0: \mu = 20000.$
2. $H_1: \mu > 20000.$
3. $\alpha = 0.05.$
4. Critical value:

σ unknown and $\text{n} = 100 > 30 \rightarrow$
use z-table at α(One-tailed test)

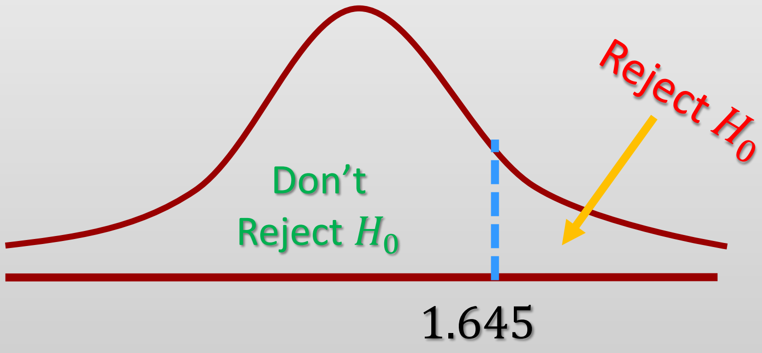
$$Z_\alpha = z_{0.05} = \frac{1.64 + 1.65}{2}$$

$$\boxed{\therefore z_\alpha = 1.645}$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−1.9	0.0287	0.0281	0.0274	0.0268	0.0[illegible]62	0.0[illegible]56	0.0250	0.0244	0.0239	0.0233
−1.8	0.03.59	0.0351	0.0344	0.0336	0.0[illegible]29	0.0[illegible]22	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0[illegible]09	0.0[illegible]01	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0591	0.0582	0.0571	0.0559

$0.0505 - 0.05 = 0.0005$ ✓✓
$0.05 - 0.0495 = 0.0005$ ✓✓

5. Critical Region: $\boxed{H_1: \mu > 20000, \text{ Right-tailed test}}$



6. Compute the test statistic value (Z):

σ unknown and $n > 30$, thus we compute:

$$Z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{23500 - 20000}{\frac{3900}{\sqrt{100}}} = 8.97$$

7. Decision:

Since $Z = 8.97$ falls in the rejection region, we reject H_0 and conclude that the average kilometers of automobile is more than 20000 ($\mu > 20000$).

Sheet (7)

7. Test the hypothesis that the average content of containers of a particular lubricant is 10 litters if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1,9.8, 9.9, 10.4, 10.3, and 9.8 litters. Use a 0.01 level of significance and assume that the distribution of contents is normal.

Solution

$$\mu_0 = 10, \qquad n = 10,$$

$$\bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{10.2 + 9.7 + \cdots + 9.8}{10} = 10.06$$

$$\therefore \bar{x} = \mathbf{10.06}$$

$$s = \sqrt{\frac{\sum_{i=}^{10}(x_i - 10.06)^2}{n-1}} = \sqrt{\frac{(10.2 - 10.06)^2 + \cdots + (9.8 - 10.06)^2}{9}}$$

$$= \sqrt{0.0604444} = 0.246$$

$$\therefore \boldsymbol{s} = 0.246$$

1. H_0: $\mu = 10$.
2. H_1: $\mu \neq 10$.
3. $\alpha = 0.01$.
4. Critical value:

σ unknown and n = 10 < 30 →

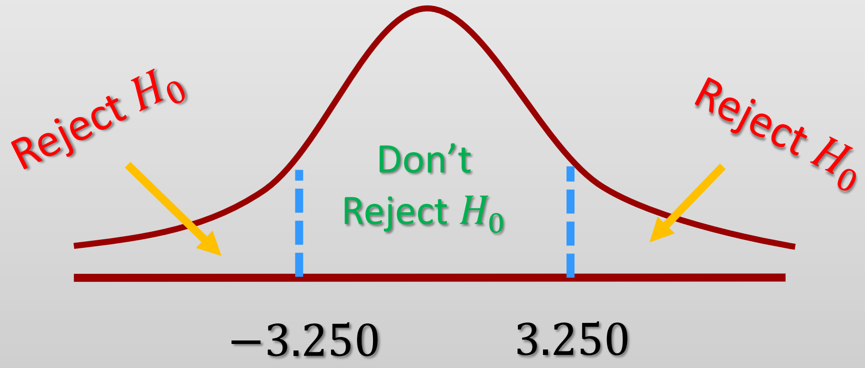
use t-table at $\frac{\alpha}{2}$, $v = n - 1$(Two-tailed test)

$$t_{\frac{\alpha}{2},\,v} = t_{\frac{\alpha}{2},\,n-1} = t_{\frac{0.01}{2},\,10-1} = t_{0.005,\,9}$$

$$\boxed{\therefore\ t_{\frac{\alpha}{2},v} = 3.250}$$

	α						
v	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.894	21.205	31.821	42.433	63.656	127.321	636.578
2	4.849	5.643	6.965	8.073	9.925	14.089	31.600
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318

5. Critical Region: $H_1: \mu \neq 10$, two−tailed test



6. Compute the test statistic value (T):

σ unknown and $n < 30$, thus we compute:

$$T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{10.06 - 10}{\frac{0.246}{\sqrt{10}}} = 0.77$$

7. Decision:

Since T $= 0.77$ falls in the acceptance region, we not reject H_0 and conclude that the average litters is 10 ($\mu = 10$).

Sheet (7)

8. According to a dietary study, a high sodium intake may be related to ulcers, stomach cancer, and migraine headaches. The human requirement for salt s only 220 milligrams per day, which is surpassed in most single servings of ready-to-eat cereals. If a random sample of 20 similar servings of certain cereal has a mean sodium content of 244 milligrams and a standard deviation of 24.5 milligrams, does this suggest at the 0.05 level of significance **that the average sodium content for a single serving of such cereal is greater than 220 milligrams**? Assume the distribution of sodium contents to be normal.

Solution

$$\mu_0 = 220, \qquad n = 20, \qquad \overline{x} = 244, \qquad s = 24.5$$

1. H_0: $\mu = 220.$
2. H_1: $\mu > 220.$
3. $\alpha = 0.05.$
4. Critical value:

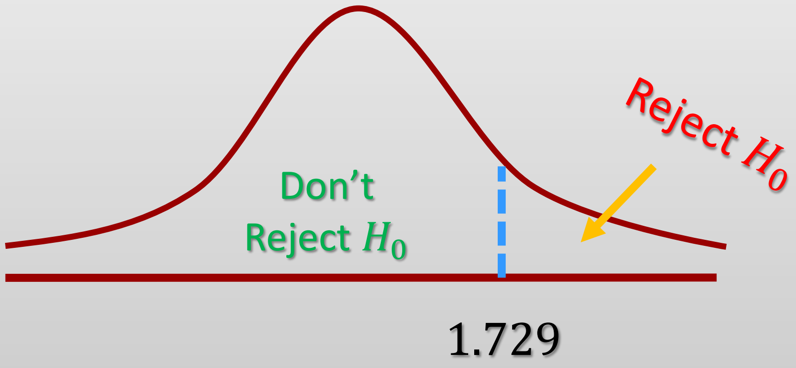
σ unknown and n = 20 < 30 →
use t-table at α, $v = n - 1$(One-tailed test)

$$t_{\alpha, v} = t_{\alpha, n-1} = t_{0.05,\, 20-1} = t_{0.05,\, 19}$$

	α						
v	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080

$$\therefore t_{\alpha,v} = 1.729$$

5. Critical Region: $H_1: \mu > 220$, Right−tailed test



6. Compute the test statistic value (T):

σ unknown and $n < 30$, thus we compute:

$$T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{244 - 220}{\frac{24.5}{\sqrt{20}}} = 4.381$$

7. Decision:

Since T $= 4.381$ falls in the rejection region, we reject H_0 and conclude that the human requirement for salt is greater than 220 milligrams per day ($\mu > 220$).