The T-Distribution

Degree of Freedom (df) or (*v*)

- Refers to the maximum number of logically independent values, which are values that have the freedom to vary, in the data sample.

X1	X2	X3	X4	X5	Mean
3	8	5	4	**10**	**6**

$$\mathrm{D_f} = N - 1$$

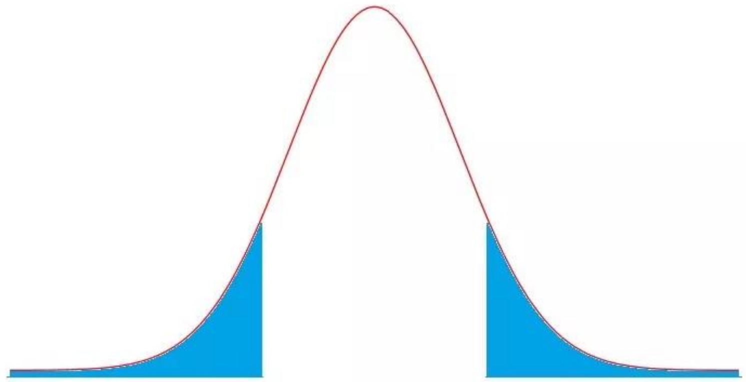
where:

$\mathrm{D_f}$ = degrees of freedom

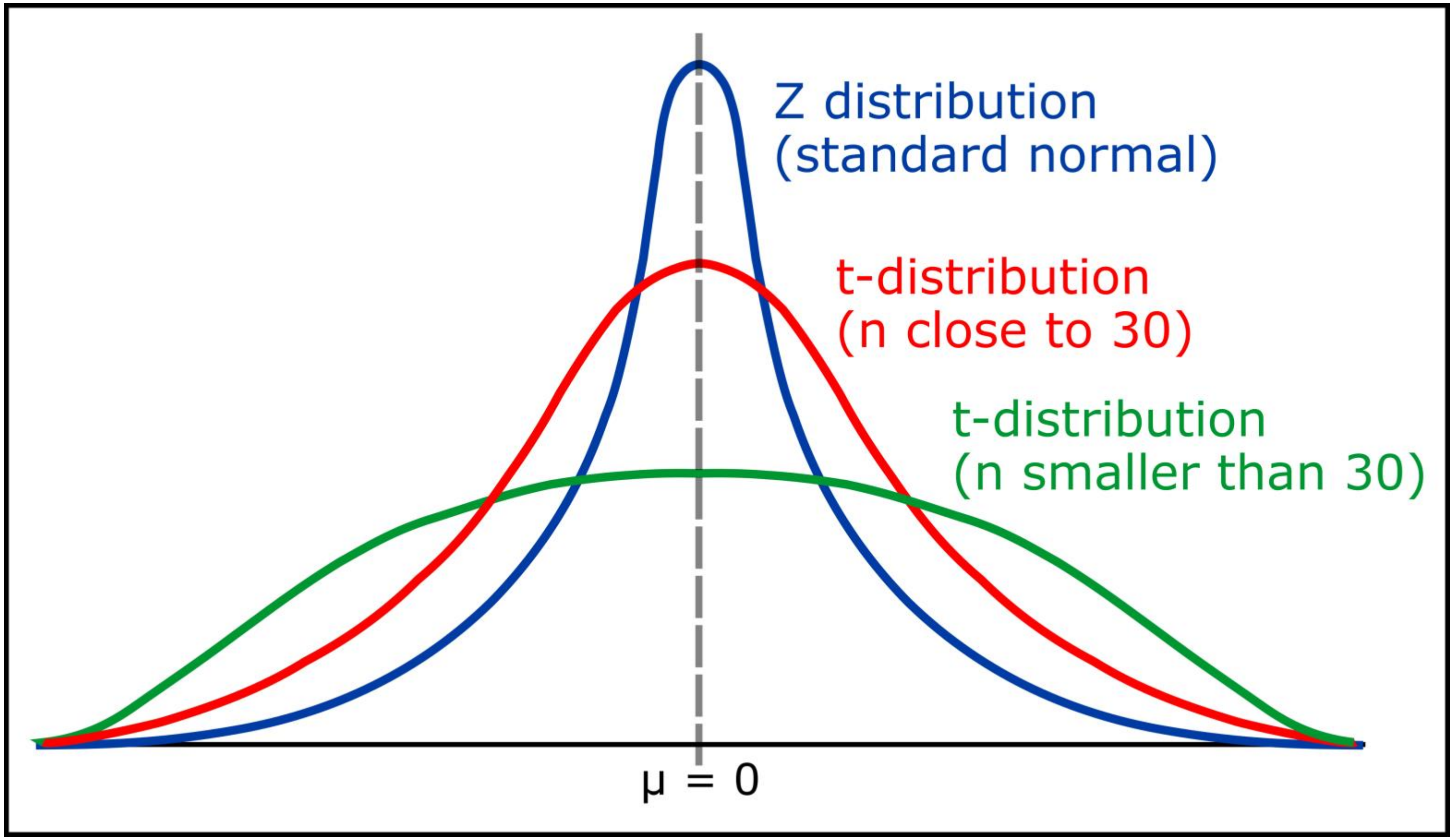
N = sample size

t-Distribution:

- The T distribution, also known as the Student's t-distribution, is a type of probability distribution that is similar to the normal distribution with its bell shape but has heavier tails.
- T distributions have a greater chance for extreme values than normal distributions, hence the fatter tails.



t-Distribution:



t-Distribution:

- Recall that, if $X_1, X_2, \ldots, X_n$ is a random sample of size n from a normal distribution with mean μ and variance σ^2, i.e. $N(\mu, \sigma)$, then

$$Z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0,1)$$

- We can apply this result only when σ^2 is known!
- If σ^2 is unknown, we replace the population variance σ^2 with the sample variance S^2

$$S^2 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}$$

to have the following statistic

$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

Result:

- If $X_1, X_2, \ldots, X_n$ is a random sample of size $n$from a normal distribution with mean μ and variance σ^2, i.e. $N(\mu, \sigma)$, then the statistic

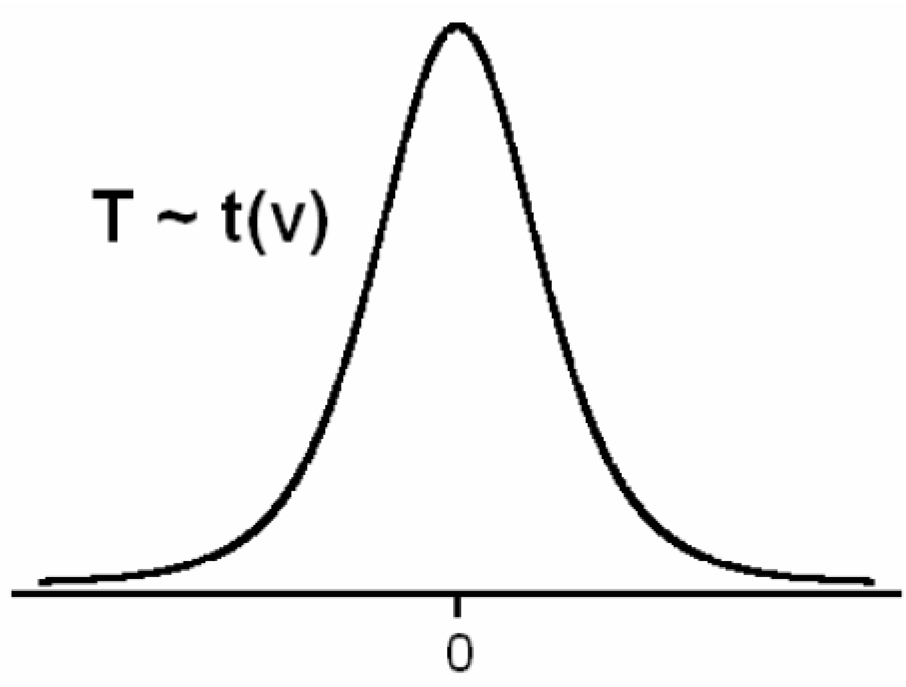
$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

- has a t-distribution with $\nu = n - 1$ degrees of freedom (df), and we write

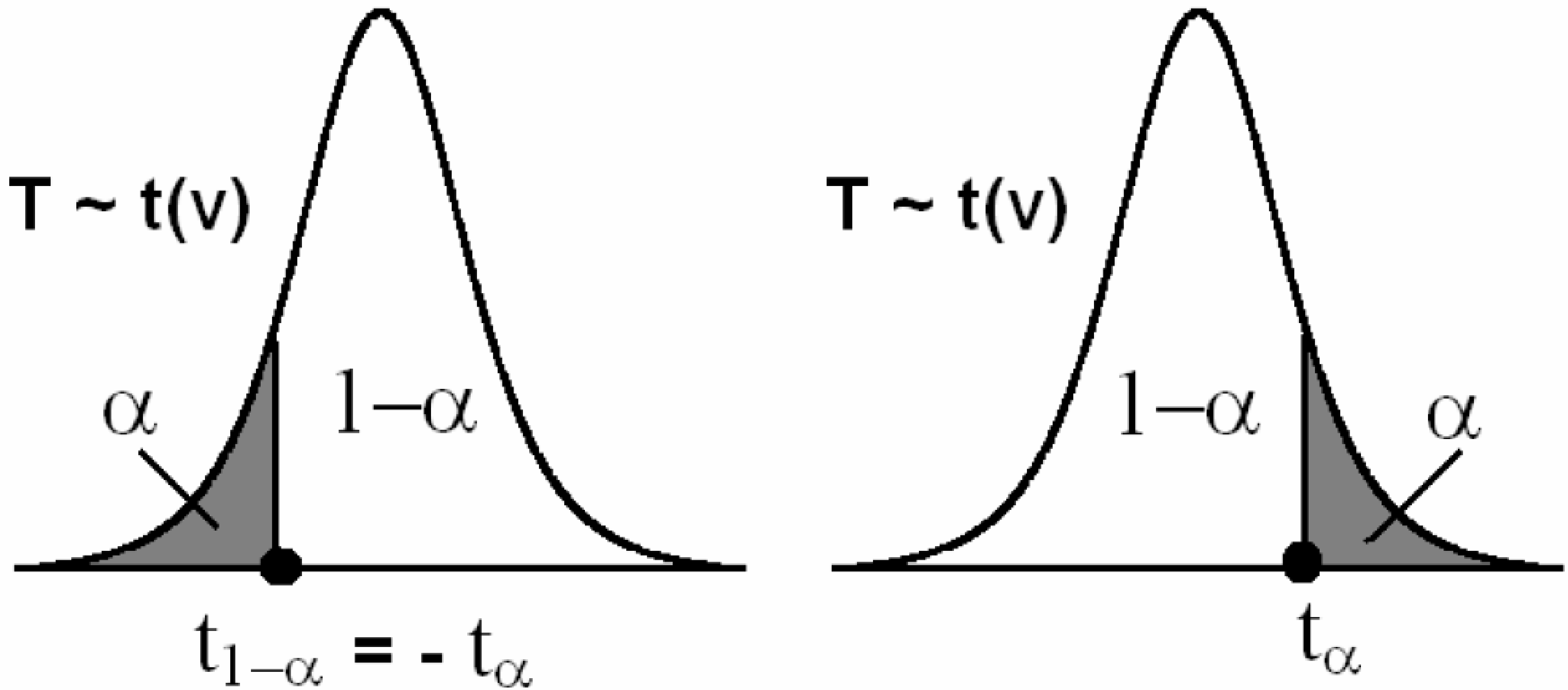
- $T \sim t(\nu)\ or\ T \sim t(n-1)$.

Note:

- t-distribution is a continuous distribution.
- The shape of t-distribution is similar to the shape of the standard normal distribution.



Notation:



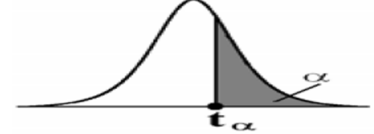
$P(T > t_\alpha) = \alpha$

- t_α = The t-value above which we find an area equal to α, that is $P(T > t_\alpha) = \alpha$
- Since the curve of the pdf of $T \sim t(\nu)$ is symmetric about 0, we have

$$t_{1-\alpha} = -t_\alpha$$

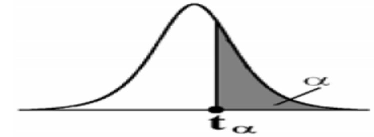
- Values of t_α are tabulated

Critical Values of the t-distribution (t_α)



ν	α = 0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Critical Values of the t-distribution (t_α)



ν	α						
	0.02	**0.015**	**0.01**	**0.0075**	**0.005**	**0.0025**	**0.0005**
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.125	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291

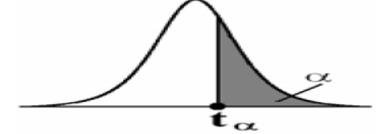
Example:

Find the t-value with $\nu = 14$ (df) that leaves an area of:

(a) 0.95 to the left.

(b) 0.95 to the right.

Critical Values of the t-distribution (t_α)



	α						
ν	**0.40**	**0.30**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**
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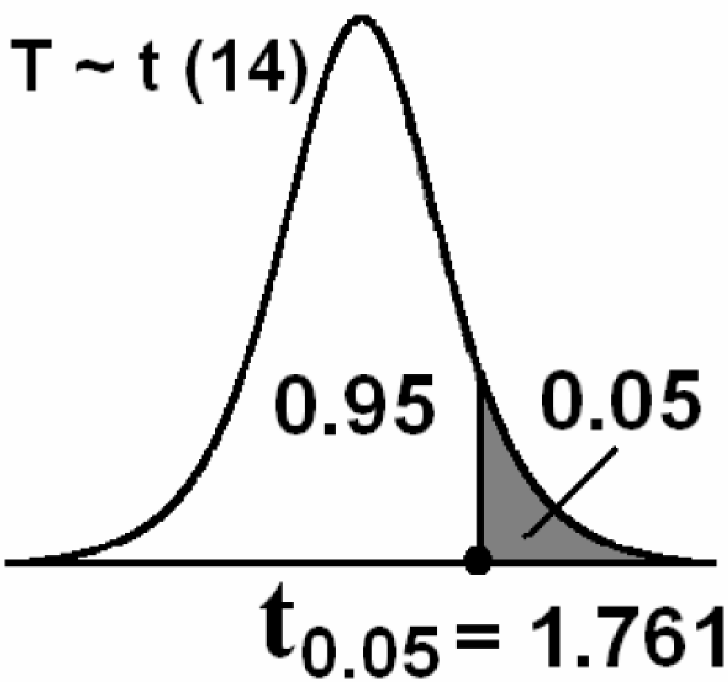
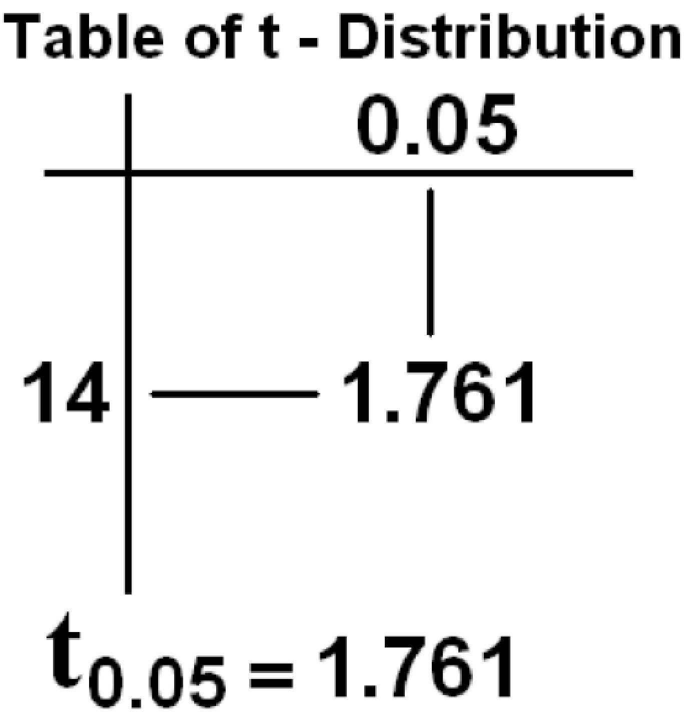
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Solution:

$$\nu = 14\,(df);\ T \sim t(14)$$

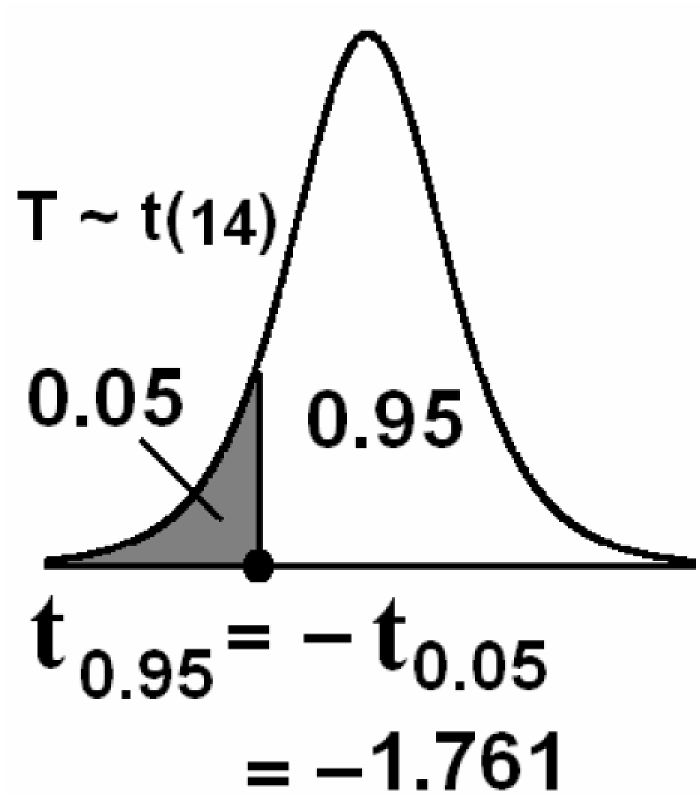
(a) The t-value that leaves an area of 0.95 to the left is $t_{0.05} = 1.761$

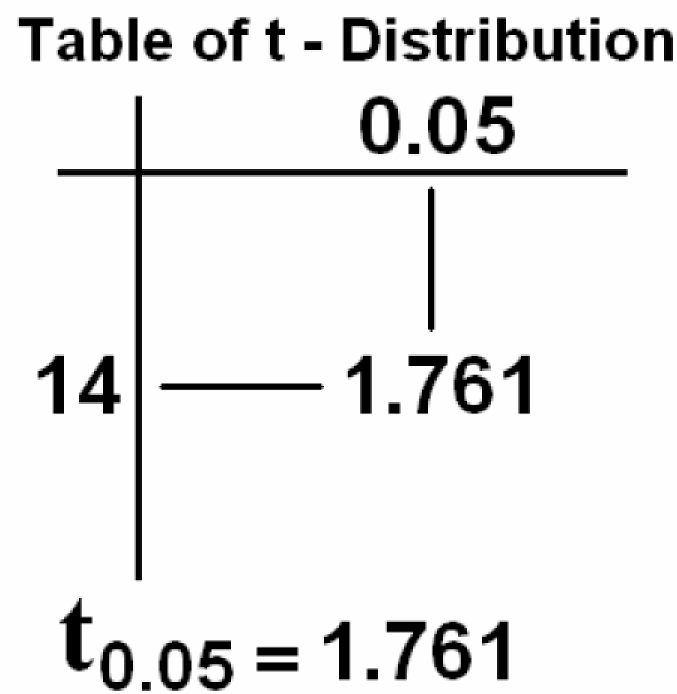




(b) The t-value that leaves an area of 0.95 to the right is

$$t_{0.95} = -\,t_{1-0.95} = -\,t_{0.05} = -\,1.761$$





Example:

For $\nu = 10$ degrees of freedom (df), find $t_{0.10}$ and $t_{0.85}$.

Solution:

$t_{0.10} = 1.372$

$t_{0.85} = -t_{1-0.85} = -t_{0.15} = -1.093$ ($t_{0.15} = 1.093$)

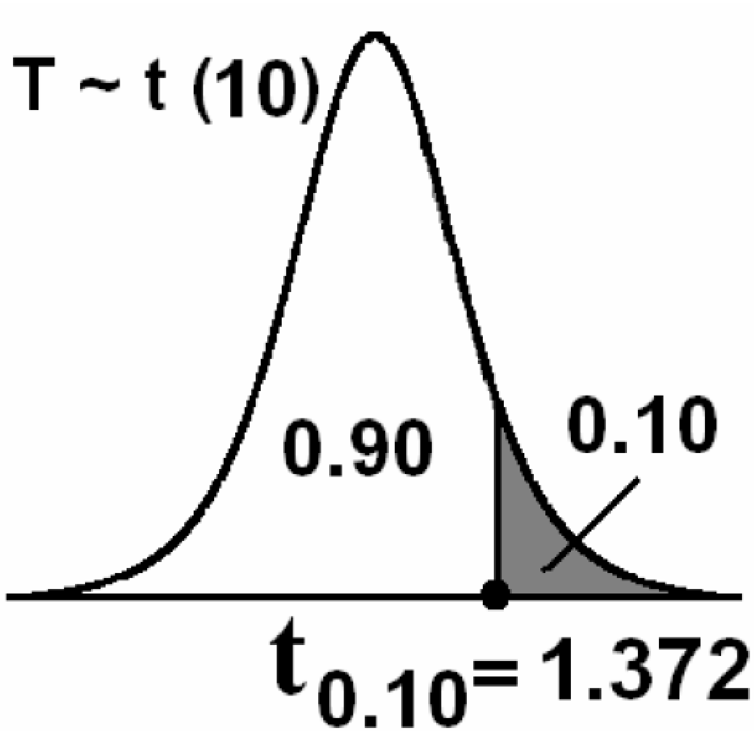


Table of t - Distribution

	0.15	0.10
10	1.093	1.372