

# The Normal Distribution

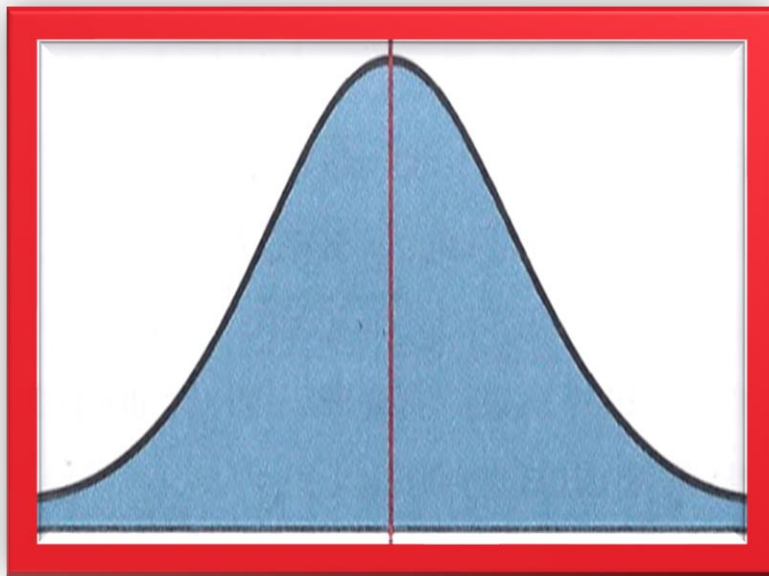
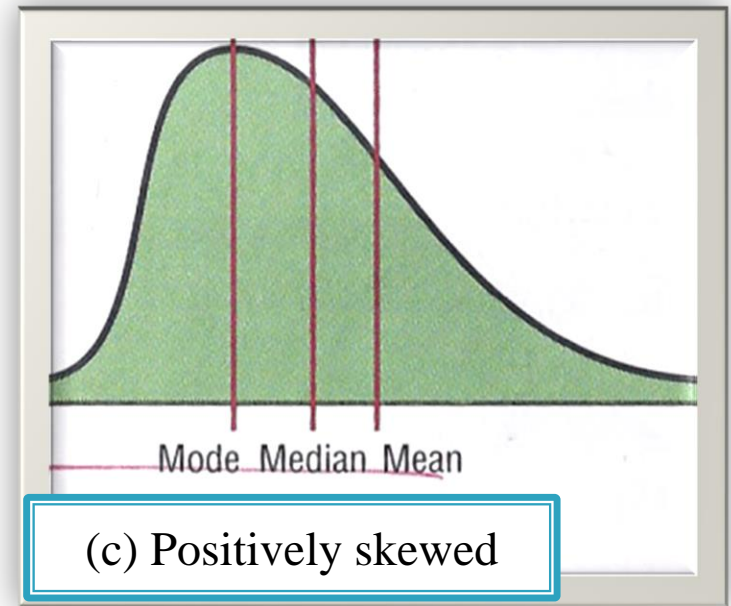
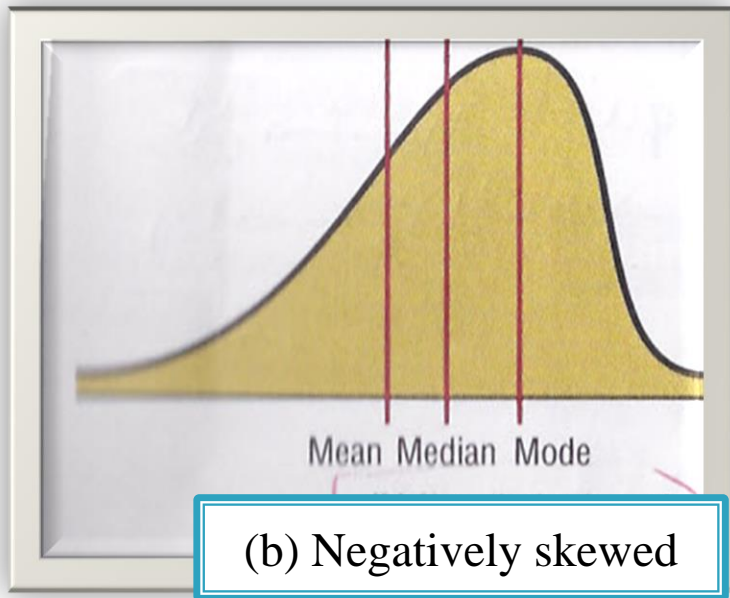
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# Introduction

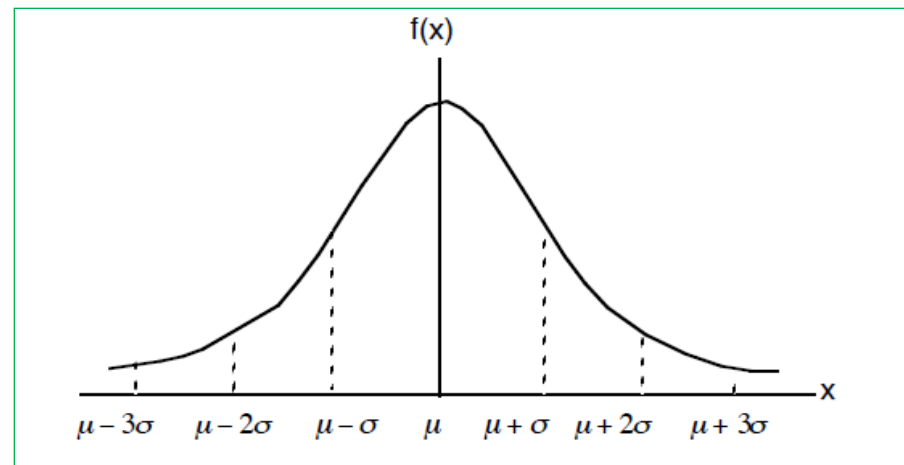
- Normal Distribution.
- Applications of the Normal Distribution.

# The Normal Distribution

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(a) Normal  
Mean = Median = Mode



□ **A normal distribution** is a continuous, symmetric, bell shaped distribution of a variable.

□ A normal distribution curve depend on two parameters .

$\mu$  —————> Position parameter

$\sigma$  —————> shape parameter

*The mathematical equation for the normal distribution:*

$$y = \frac{e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$$

*where*

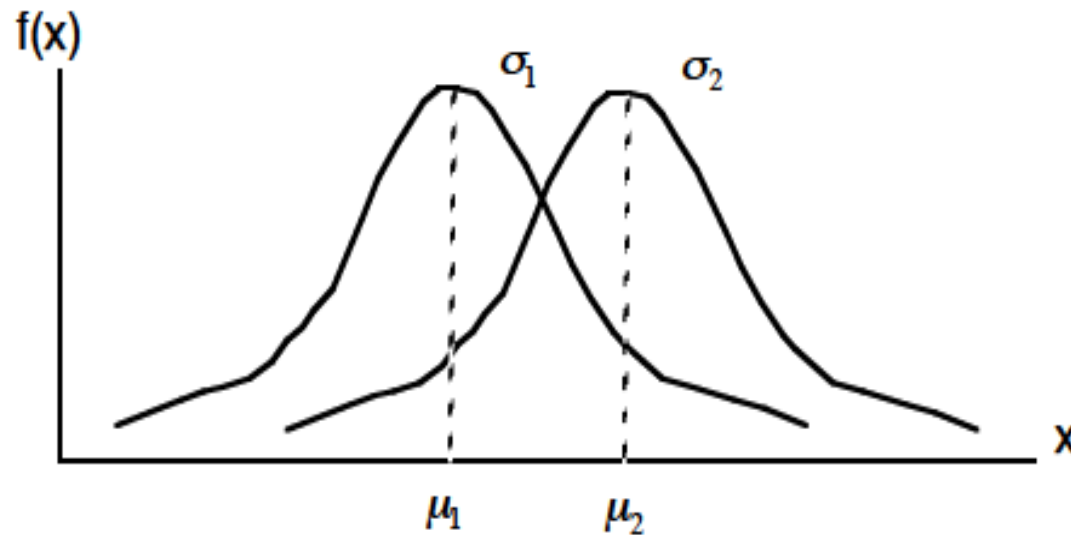
$e \approx 2.718$

$\pi \approx 3.14$

$\mu \approx$  *population mean*

$\sigma \approx$  *population standard deviation*

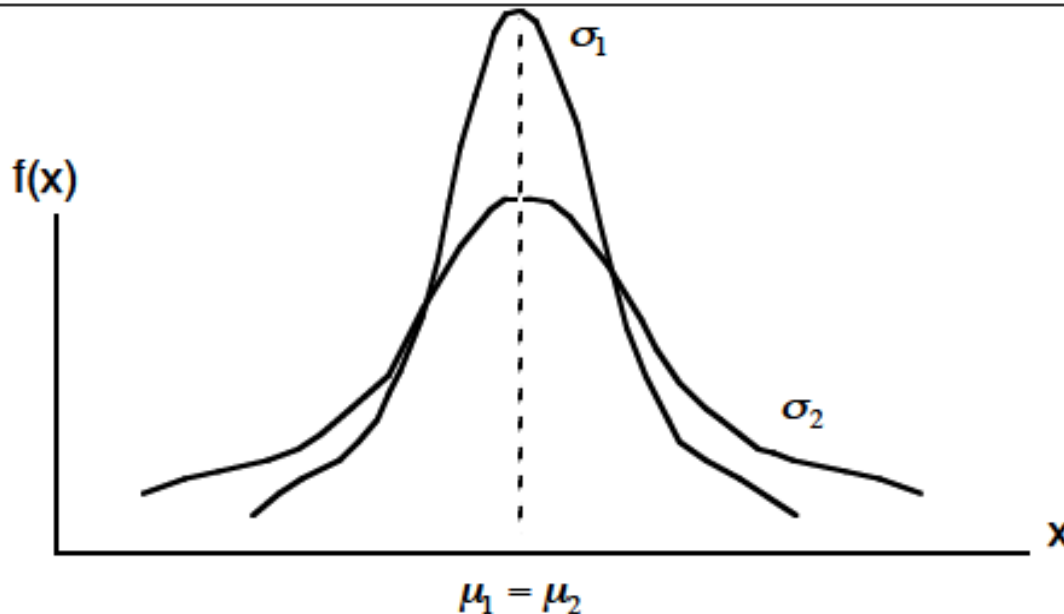
1



Normal curves  
with  $\mu_1 \neq \mu_2$  and  
 $\sigma_1 = \sigma_2$

(1) Different  
means but same  
standard  
deviations.

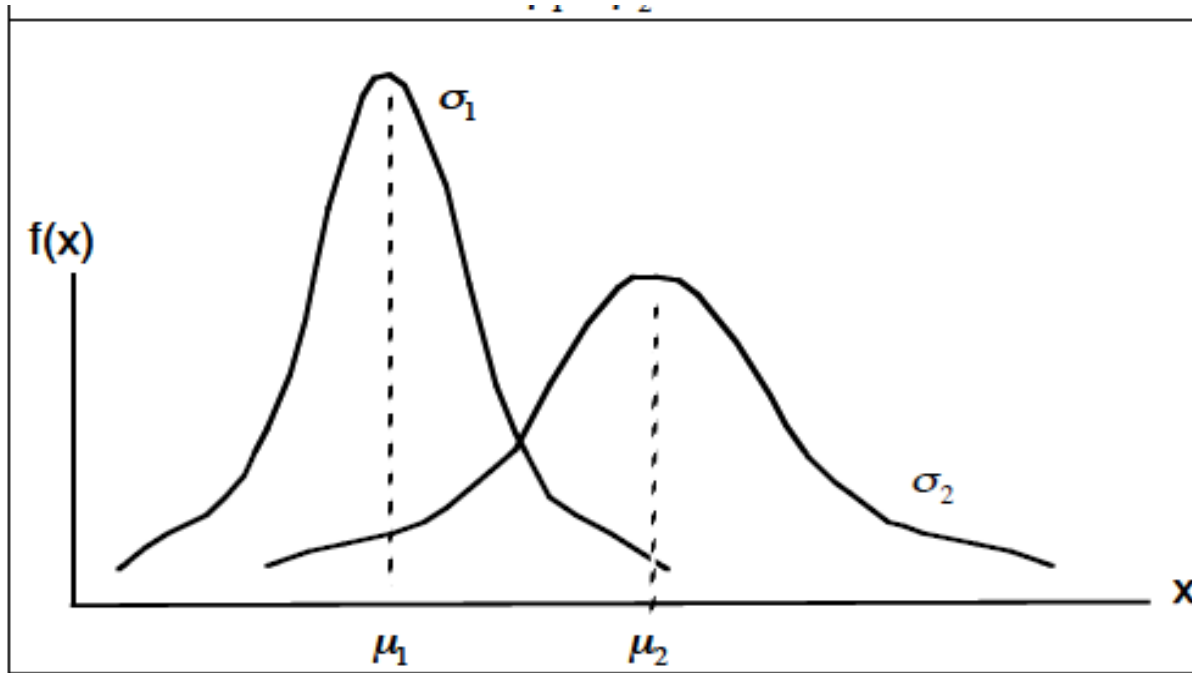
2



Normal curves  
with  $\mu_1 = \mu_2$  and  
 $\sigma_1 < \sigma_2$

(2) Same means  
but different  
standard  
deviations .

3

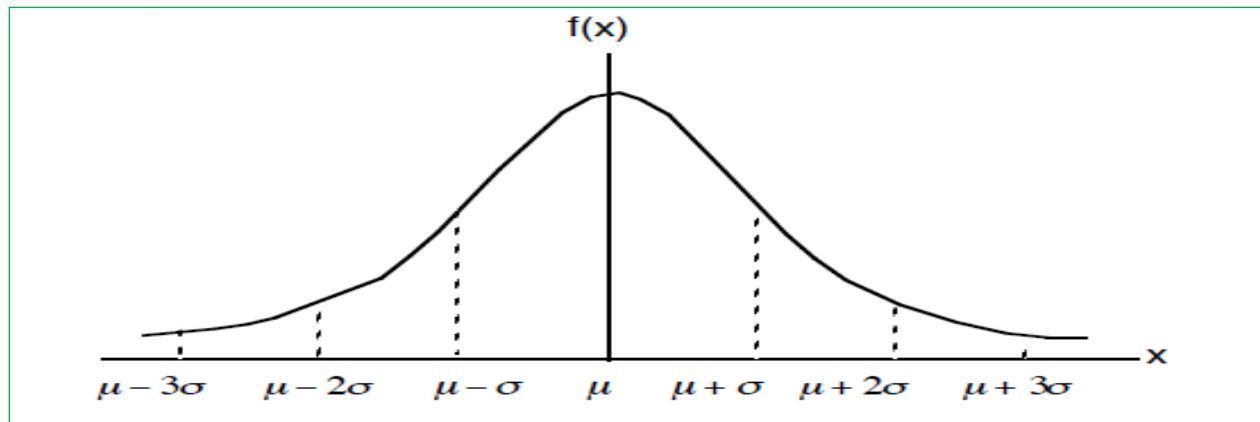


Normal curves  
with  $\mu_1 < \mu_2$  and  
 $\sigma_1 < \sigma_2$

(3) Different  
means and  
different standard  
deviations .



# Properties of the Normal Distribution



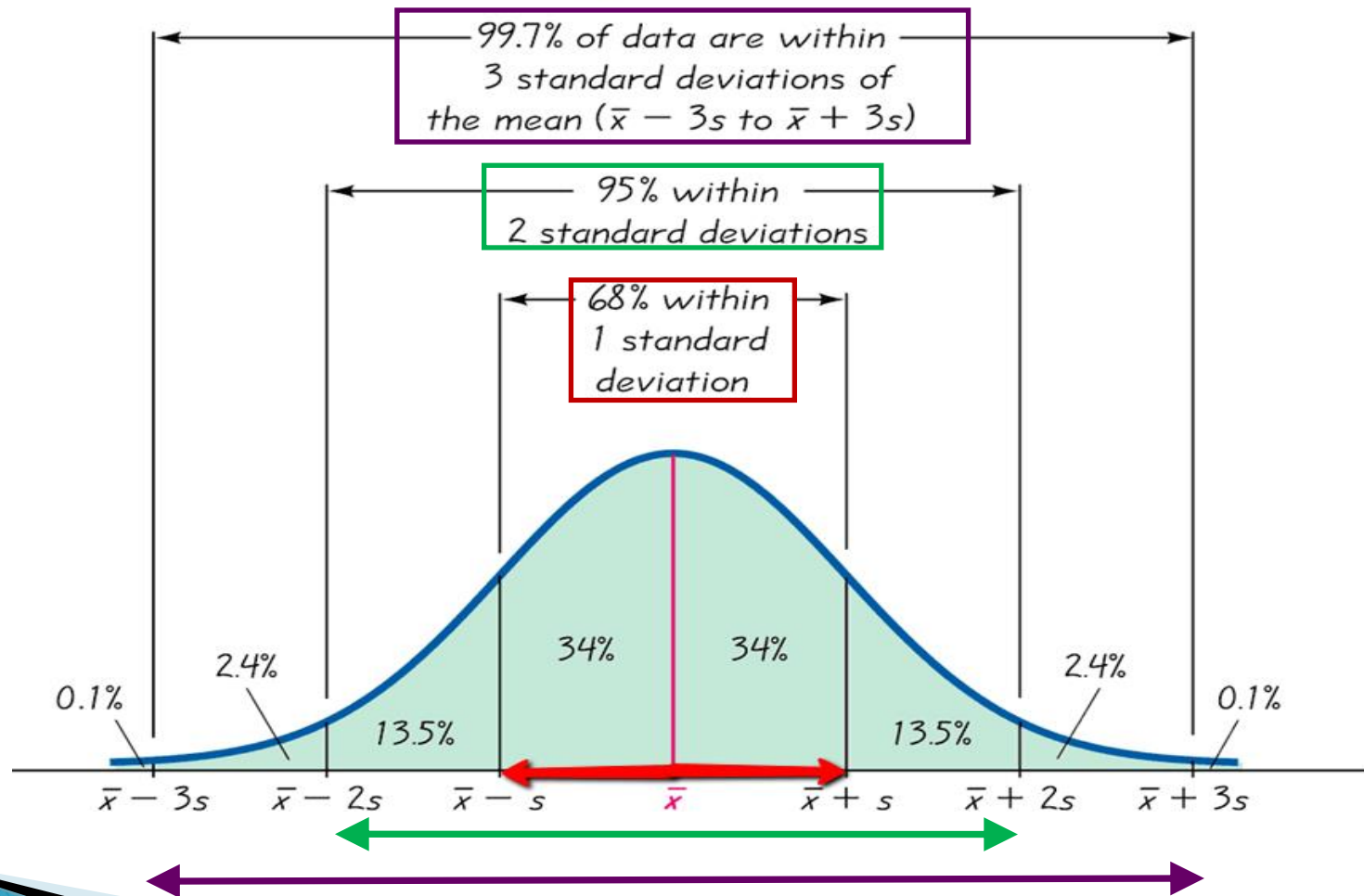
- The normal distribution curve is **bell-shaped**.
- The mean, median, and mode are **equal** and located at the center of the distribution.
- The normal distribution curve is **unimodal** (single mode).
- The curve is **symmetrical** about the mean.
- The curve is **continuous**.
- The curve **never touches the x-axis**.
- The total area under the normal distribution curve is **equal to 1 or 100%**.

□ The area under the normal distribution curve that lies within

- **one** standard deviation of the mean is approximately 0.68 (68%). The interval between  $(\bar{x} - s, \bar{x} + s)$
- **two** standard deviations of the mean is approximately 0.95 (95%). The interval between  $(\bar{x} - 2s, \bar{x} + 2s)$
- **three** standard deviations of the mean is approximately 0.997 (99.7%). The interval between  $(\bar{x} - 3s, \bar{x} + 3s)$

# Empirical Rule: Normal Distribution

## (Areas Under the Normal Curve)



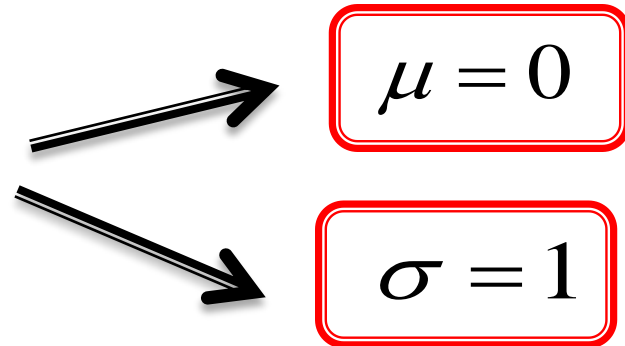
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# The Standard Normal Distribution

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□ The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1.

□ The formula for the standard normal distribution is

$$y = \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}$$


The diagram shows the formula for the standard normal distribution,  $y = \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}$ . Two arrows originate from the right side of the formula. The upper arrow points to a red-bordered box containing the equation  $\mu = 0$ . The lower arrow points to another red-bordered box containing the equation  $\sigma = 1$ .

□ All Normal Distribution can be transformed into standard Distribution.

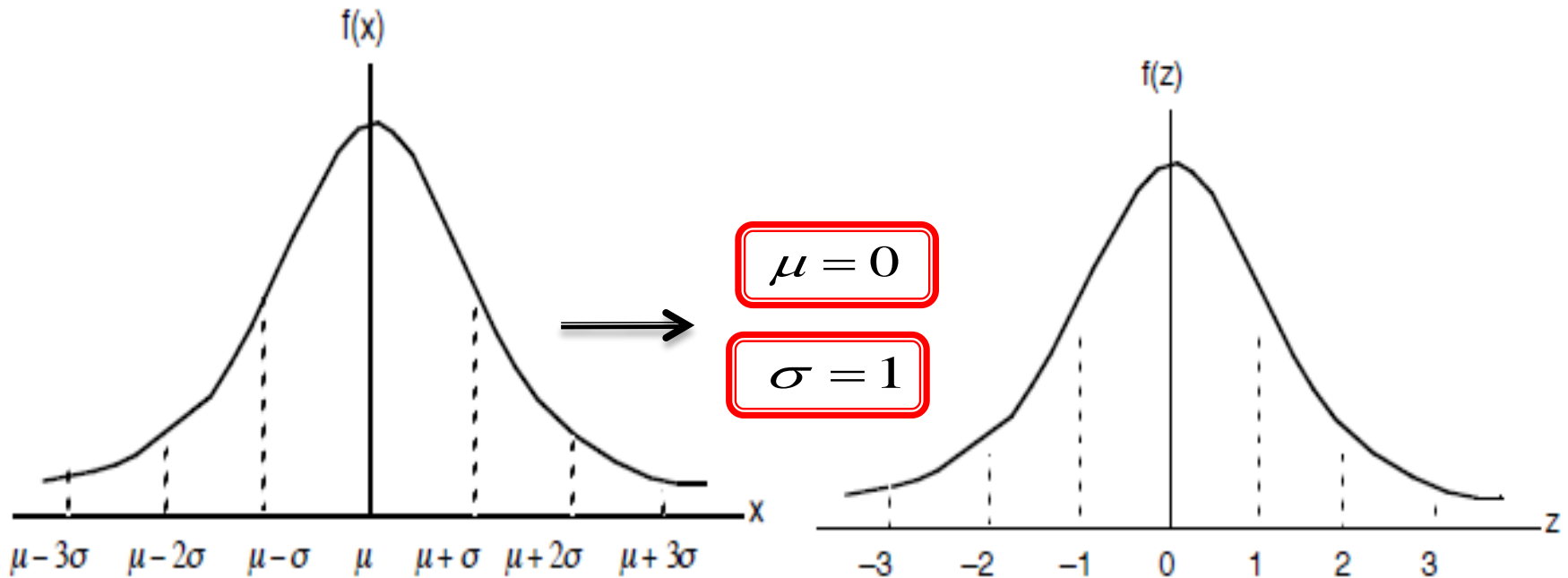
$$z = \frac{\textit{value} - \textit{mean}}{\textit{standard deviation}}$$

or

$$z = \frac{x - \mu}{\sigma}$$

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# Empirical Rule: Standard Normal Distribution



Normal Distribution Curve

Standard Normal Distribution Curve

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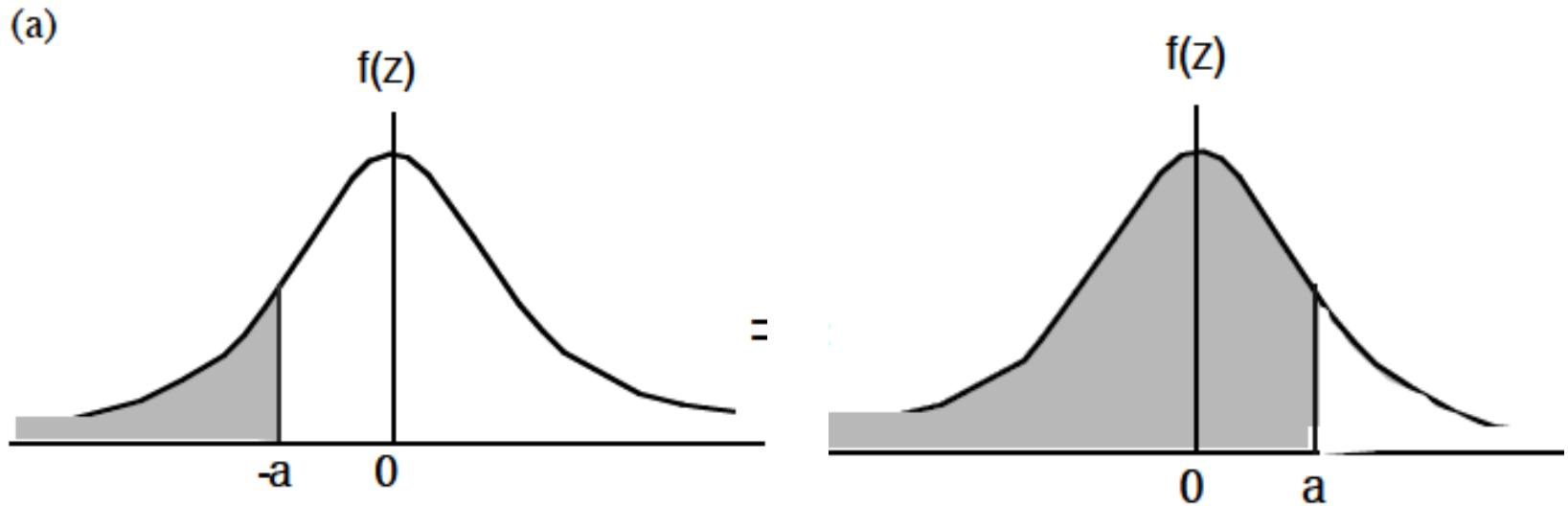
□ The area under the standard normal distribution curve that lies within

- **one** standard deviation of the mean is approximately 0.68 (68%). The interval between (-1,1).
- **two** standard deviations of the mean is approximately 0.95 (95%). The interval between (-2,2).
- **three** standard deviations of the mean is approximately 0.997 (99.7%). The interval between (-3,3).



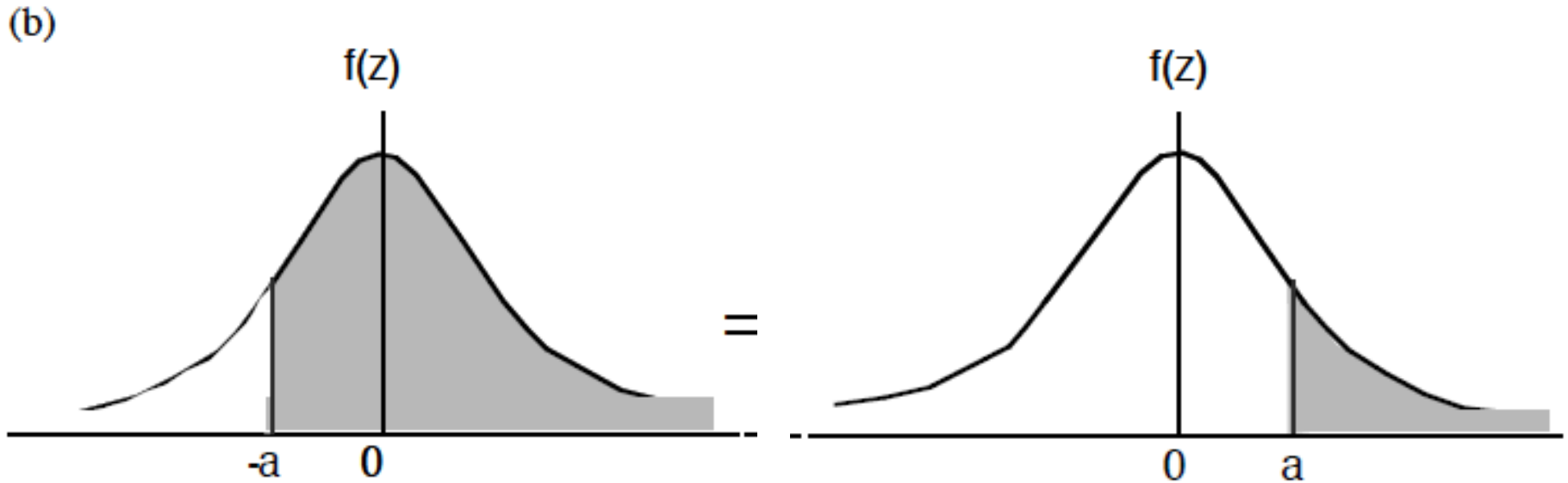
# Finding Areas Under the Standard Normal Distribution Curve:

1. To the left of any  $Z$  value



$$P(z < -a) = P(z > a) = Q(a)$$

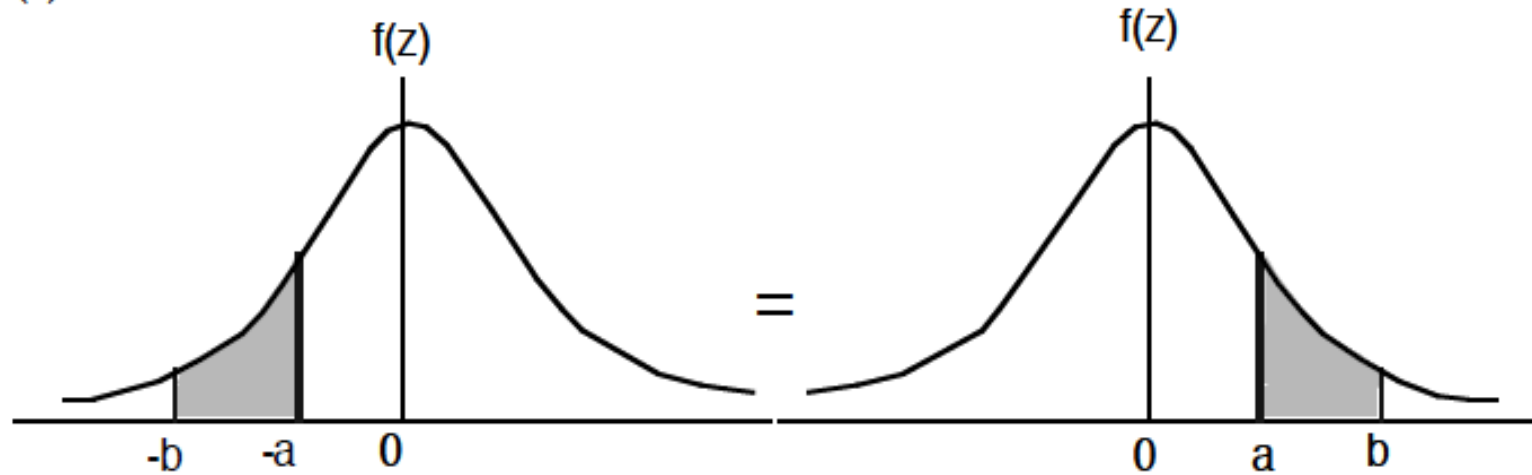
## 2.To the right of any Z value



$$P(z > -a) = P(z > a) = 1 - Q(a)$$

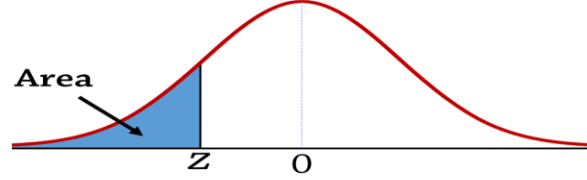
### 3. Between any two Z values

(c)



$$\begin{aligned} P(-b < z < -a) &= P(a < z < b) = P(z > a) - P(z > b) \\ &= Q(a) - Q(b) \end{aligned}$$





<b>z</b>	<b>0.00</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
<b>-3.4</b>	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
<b>-3.3</b>	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
<b>-3.2</b>	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
<b>-3.1</b>	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
<b>-3.0</b>	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
<b>-2.9</b>	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
<b>-2.8</b>	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
<b>-2.7</b>	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
<b>-2.6</b>	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
<b>-2.5</b>	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
<b>-2.4</b>	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
<b>-2.3</b>	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
<b>-2.2</b>	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
<b>-2.1</b>	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
<b>-2.0</b>	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
<b>-1.9</b>	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
<b>-1.8</b>	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
<b>-1.7</b>	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
<b>-1.6</b>	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
<b>-1.5</b>	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
<b>-1.4</b>	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
<b>-1.3</b>	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
<b>-1.2</b>	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
<b>-1.1</b>	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
<b>-1.0</b>	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
<b>-0.9</b>	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
<b>-0.8</b>	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
<b>-0.7</b>	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
<b>-0.6</b>	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
<b>-0.5</b>	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
<b>-0.4</b>	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
<b>-0.3</b>	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
<b>-0.2</b>	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
<b>-0.1</b>	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
<b>-0.0</b>	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641