* Conditional Probability :

It is the probability of an event given that another event has occurred.

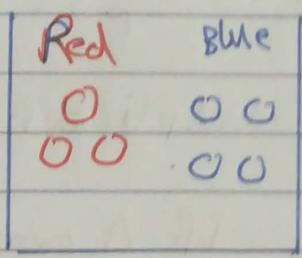
$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

A and B are "Dependent"

$$P(A \cap B) = P(A)\,P(B|A)$$

EX (11):

A Box containing 3 Red and 4 Blue balls. find the probability of getting two balls with the same Color.



→ without Replacement

$P(R_1)$: First is Red
$P(R_2)$: Second is Red
$P(B_1)$: First is Blue
$P(B_2)$: second is Blue

$P(R_1) = \frac{3}{7}$ $\qquad$ $P(R_2|R_1) = \frac{2}{6}$

$P(B_1) = \frac{4}{7}$ $\qquad$ $P(B_2|B_1) = \frac{3}{6}$

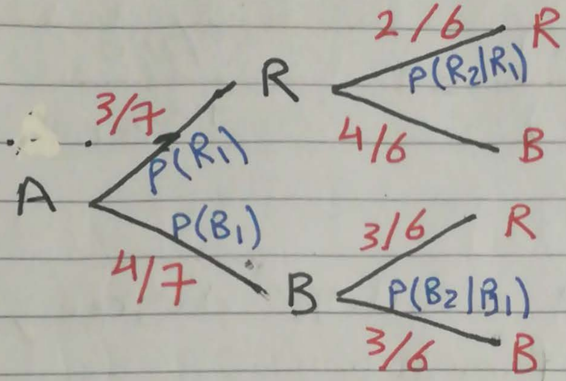
$$P(A) = P(R_1 \cap R_2) + P(B_1 \cap B_2)$$

$$= P(R_1)\,P(R_2|R_1) + P(B_1)\,P(B_2|B_1)$$

$$= \left(\frac{3}{7}\right)\left(\frac{2}{6}\right) + \left(\frac{4}{7}\right)\left(\frac{3}{6}\right) =$$

$$= \frac{6}{42} + \frac{12}{42} = \frac{18}{42} = \boxed{\frac{3}{7}}$$

* It can be solved using Tree



$P(RR \text{ or } BB)$
$= \left(\frac{3}{7}\right)\left(\frac{2}{6}\right) + \left(\frac{4}{7}\right)\left(\frac{3}{6}\right)$
$= \boxed{\frac{3}{7}}$

* with Replacement:

$P(R_1) = \frac{3}{7}$ $\qquad$ $P(R_2|R_1) = P(R_1) = \frac{3}{7}$

$P(B_1) = \frac{4}{7}$ $\qquad$ $P(B_2|B_1) = P(B_1) = \frac{4}{7}$

$P(A) = P(R_1)\,P(R_1) + P(B_1)\,P(B_1) =$
$= \left(\frac{3}{7}\right)\left(\frac{3}{7}\right) + \left(\frac{4}{7}\right)\left(\frac{4}{7}\right) = \frac{9}{49} + \frac{16}{49}$
$= \boxed{\frac{25}{49}}$

* For independent events $P(B|A) = P(B)$
$P(A \cap B) = P(A)\,P(B)$

Ex 12:

Education level	Male	Female	Total
Elementary	38	45	83
Secondary	28	50	78
College	22	17	39
Total	88	112	200

If a person is selected randomly from this group, Find the probability that:
a) A person is a male, given that he has a secondary school.

$$P(m \mid s) = \frac{P(m \cap s)}{P(s)} = \frac{28/200}{78/200}$$

$$= \boxed{\frac{28}{78}}$$

b) A person doesn't have a college degree, given that she is a female.

لاحظ هنا شوف اللي مش college

$$P(\bar{C} \mid F) = \frac{P(\bar{C} \cap F)}{P(F)} = \frac{95/200}{112/200} = \boxed{\frac{95}{112}}$$

Another solution

$$P(\bar{C} \cap F) = P(F) - P(C \cap F) = \frac{112}{200} - \frac{17}{200}$$

$$= \boxed{\frac{95}{200}}$$ وكمل الحل عادي

Ex (13): independent events

$P(A)$: sniper (A) $= \frac{1}{2}$ $\quad P(\bar{A}) = \frac{1}{2}$

$P(B)$: sniper (B) $= \frac{1}{4}$ $\quad P(\bar{B}) = \frac{3}{4}$

Find the probability that:

a) only one hits the target:

$$P(A \cap \bar{B}) + P(\bar{A} \cap B)$$
$$= P(A)\,P(\bar{B}) + P(\bar{A})\,P(B)$$
$$= \left(\tfrac{1}{2}\right)\left(\tfrac{3}{4}\right) + \left(\tfrac{1}{2}\right)\left(\tfrac{1}{4}\right) = \frac{3}{8} + \frac{1}{8} =$$
$$\boxed{\frac{4}{8}} = \boxed{\frac{1}{2}}$$

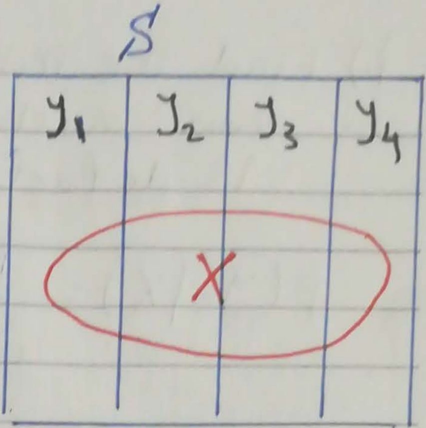
b) A and B hit the target:

$$P(A \cap B) = P(A)\,P(B) = \left(\tfrac{1}{2}\right)\left(\tfrac{1}{4}\right) = \boxed{\frac{1}{8}}$$

c) At least one sniper hits the target:

$$= P(A \cap B) + P(A \cap \bar{B}) + P(\bar{A} \cap B)$$
$$= P(A)\,P(B) + P(A)\,P(\bar{B}) + P(\bar{A})\,P(B)$$
$$= \left(\tfrac{1}{2}\right)\left(\tfrac{1}{4}\right) + \left(\tfrac{1}{2}\right)\left(\tfrac{3}{4}\right) + \left(\tfrac{1}{2}\right)\left(\tfrac{1}{4}\right) = \boxed{\frac{5}{8}}$$

* Sum Rule:



$$P(X) = P(X \cap y_1) + P(X \cap y_2) + P(X \cap y_3) + P(X \cap y_4)$$

$$P(X) = P(X|y_1)\,P(y_1) + P(X|y_2)\,P(y_2) + P(X|y_3)\,P(y_3) + P(X|y_4)\,P(y_2)$$

$$P(X|y_1) = \frac{P(X \cap y_1)}{P(y_1)}$$

$$\boxed{P(X) = \sum_{y} P(X|y)\,P(y)}$$

* Product Rule:

$$\boxed{P(X \cap y) = P(X|y)\,P(y)}$$

* Bayes Rule

$$\boxed{P(y/X) = \frac{P(X \cap y)}{P(X)} = \frac{P(X|y)\,P(y)}{P(X)}}$$

EX 14:

Machine (1) $\Rightarrow$ 30% $\quad P(y_1) = 0.3$

Machine (2) $\Rightarrow$ 45% $\quad P(y_2) = 0.45$

Machine (3) $\Rightarrow$ 25% $\quad P(y_3) = 0.25$

P(Defective) = P(X)

$P(X|y_1) = 0.02$, $P(X|y_2) = 0.03$, $P(X|y_3) = 0.02$

a) Find the probability that the product is defective.

$$P(X) = \sum_{y} P(X|y)\,P(y)$$

$$= P(X|y_1)\,P(y_1) + P(X|y_2)\,P(y_2) + P(X|y_3)\,P(y_3)$$

$$= (0.02)(0.3) + (0.03)(0.45) + (0.02)(0.25)$$

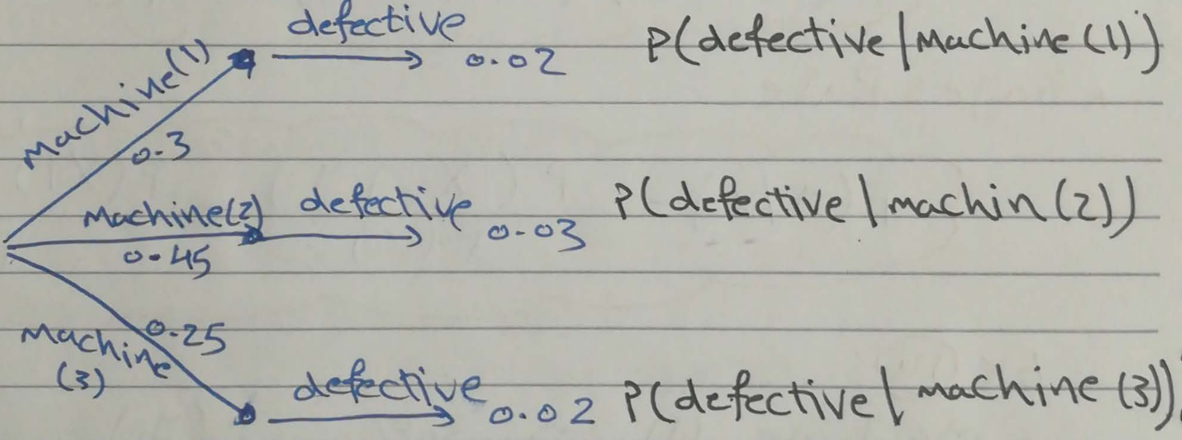
$$\boxed{P(X) = 0.0245}$$

b) Find the probability that the product is from machine (3) given that the product is defective.

$$P(y_3|X) = \frac{P(X|y_3)P(y_3)}{P(X)} = \frac{(0.02)(0.25)}{0.0245}$$

$$= \boxed{\frac{10}{49} = 0.2}$$

$$P(y|X)P(X) = P(X|y)P(y)$$



*Rare Disease Example

A test for a rare disease claims that it will report a positive result for 99.5% of people with the disease, and will report a negative result for 99.9% of those without the disease. We know that the disease is present in the population at 1 in 100,000. Knowing this information, what is the likelihood that an individual who tests given positive will actually has the disease.

$$\frac{\text{Likelihood}}{\text{Probability}}$$

$$P(+\text{test} \mid +\text{disease}) = 0.995$$
$$P(-\text{test} \mid -\text{disease}) = 0.999$$
$$P(+\text{disease}) = 0.0001$$
$$P(-\text{disease}) = 1 - 0.0001 = 0.9999$$

$$P(+\text{disease} \mid +\text{test}) = \frac{P(+\text{test} \mid +\text{disease})\, P(+\text{disease})}{P(+\text{test})}$$

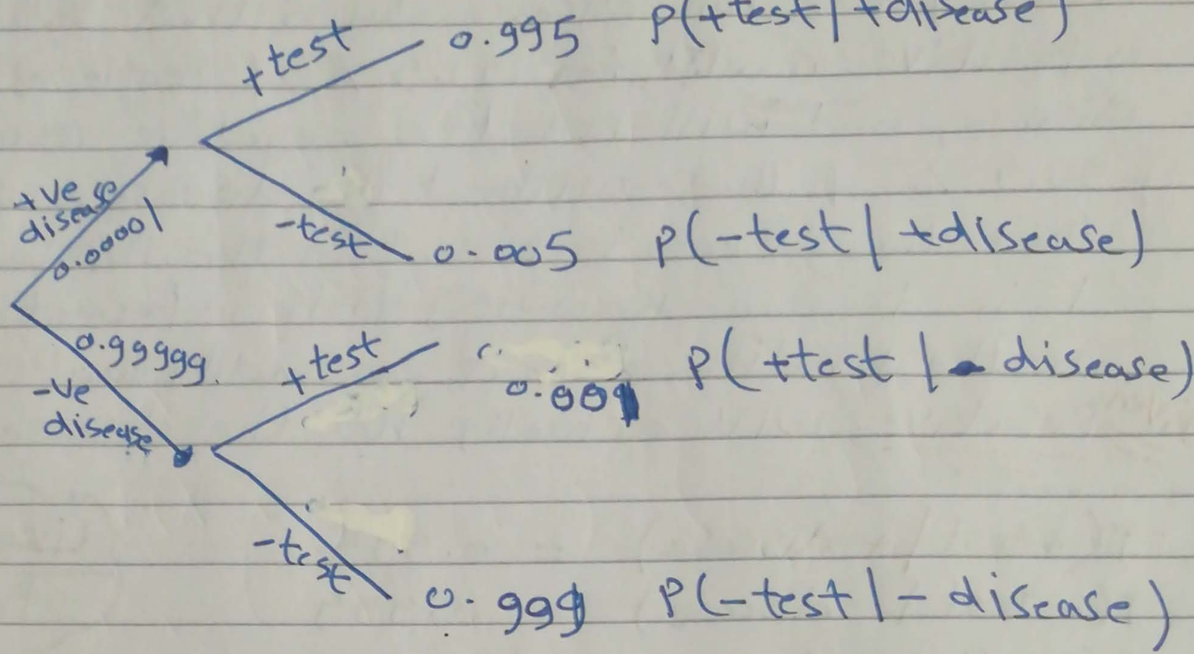
$$P(+\text{test} \mid -\text{disease}) = 1 - P(-\text{test} \mid -\text{disease})$$
$$= 1 - 0.999 = 0.001$$

$$P(+\text{test}) = \sum_{\text{disease}} P(+\text{test} \mid \text{disease})\, P(\text{disease})$$

$$= P(+\text{test} \mid +\text{disease})\, P(+\text{disease}) + P(+\text{test} \mid -\text{disease})\, P(-\text{disease})$$
$$= (0.995)(0.00001) + (0.001)(0.99999)$$

$$= 0.00100994$$

$$P(+\text{disease} \mid +\text{test}) = \frac{(0.995)(0.00001)}{0.00100994}$$
$$= 0.0099 \simeq 0.001$$


+ve disease
0.00001
0.99999
-ve disease
+test
-test
+test
-test
0.995
0.005
0.001
0.999
P(+test | +disease)
P(-test | +disease)
P(+test | - disease)
P(-test | - disease)