Probability

Lectur 01

It is the study of outcomes of random Phenomena or "experiment".

* **Independent Events:**
  The outcome of a certain event is not affected by any other event.
* **Sample space:**
  All Possible outcomes of a random Phenomena or "experiment"

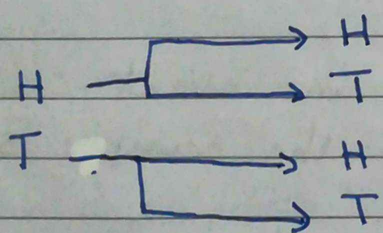
EX ①:

Experiment	Sample space (S)	n(S)
Tossing a coin	{H, T}	2
Rolling a Die	{1, 2, 3, 4, 5, 6}	6
True or false question	{T, F}	2

EX ②:

Find sample space for tossing a coin twice.

First outcome — Second outcome — Sample space



Sample space:

H H
H T
T H
T T

n(S) = 4

EX ③:

Find sample space for rolling a die twice

Dice (1) — Dice (2)

n(S) = 36

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	⋮					
⋮						
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

If N : No. of all Possible outcomes
n : No. of trials
$n(S)$: No. of sample space

$$n(S) = (N)^n$$

* Event : (E)
It is a subset of the sample space, it is a particular outcome. $E \subset S$

EX (4):
Getting an odd number when rolling a die.
$S = \{1, 2, 3, 4, 5, 6\}$ $n(S) = 6$
$E = \{1, 3, 5\}$
$n(E) = 3$

EX (5):
Getting at least one Head when tossing a coin twice.
$S = \{(H,H), (H,T), (T,H), (T,T)\}$
$E = \{(H,H), (H,T), (T,H)\}$
$n(S) = 4$ $n(E) = 3$

* Probability of an event: "Equally likely events"

$$P(E) = \frac{n(E)}{n(S)}$$

Where:
$n(E)$: No. of elements in the event.
$n(S)$: No. of elements in the sample space.

EX (6): "Rolling a die"
a) $P(\text{getting "1"}) = \frac{1}{6}$

b) $P(\text{getting} > 2)$
$E = \{3, 4, 5, 6\}$ $n(E) = 4$ $n(S) = 6$

$$P = \frac{4}{6} = \frac{2}{3}$$

* Basic Probability Rules:

*** Rule ①:**

$$0 \leq P(A) \leq 1$$

Impossible event (ϕ) — $P(\phi) = 0$

Certain or sure event — $P = 1$

*** Rule ②:**

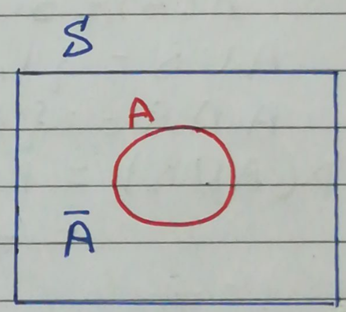
Sum of all probabilities in the sample space is **one**.

* Types of events:

1] Impossible events: (ϕ)

$$P(\phi) = 0$$

2] Complement of an event: ($\bar{E}$)



$$P(\bar{E}) = 1 - P(E)$$

EX ⑦:

Probability of getting number greater than (4) when rolling a die once.

$$P(E) = \frac{2}{6} = \frac{1}{3}$$

Probability of getting number less than (4)

it is the complementary of $P(\bar{E})$

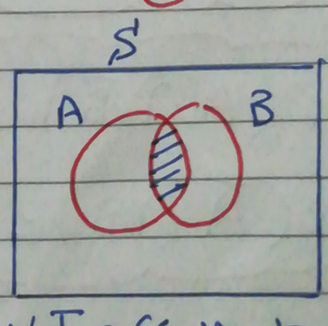
$$P(\bar{E}) = 1 - P(E) = 1 - \frac{1}{3} = \frac{2}{3}$$

3] Intersection of events:

($A \cap B$) (A and B)

احتمال وقوع (A, B) في نفس الوقت

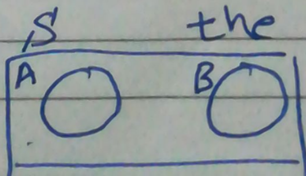
$$P(A \cap B)$$



4] mutually exclusive or Disjoint events

(A, B) can NOT occur at the same time.

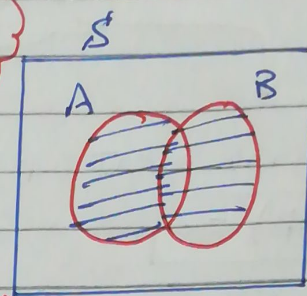
$$P(A \cap B) = 0$$



5] Union of events : "Addition Rule"

$(A \cup B)$ $(A \text{ or } B)$ "At least one event occur"

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



If A and B are mutually exclusive or disjoint events.

$P(A \cap B) = 0$

$$P(A \cup B) = P(A) + P(B)$$

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"disjoint"

Ex (8):

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 4, 5\}$ $B = \{4, 5, 6, 7, 8\}$

$n(A) = 5$ $n(B) = 5$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A \cap B = \{4, 5\}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{5}{10} + \frac{5}{10} - \frac{2}{10} = \frac{8}{10} = 0.8$$

6] Difference between two events :

"one event only" "A only" or "B only"

$$P(A - B) = P(A) - P(A \cap B)$$
$$P(B - A) = P(B) - P(A \cap B)$$

$$P(A \cap \bar{B}) = P(A - B) = P(A) - P(A \cap B)$$

$$P(\bar{A} \cap B) = P(B - A) = P(B) - P(A \cap B)$$

Ex (9):

$P(\bar{A}) = 0.3$, $P(B) = 0.4$, $P(A \cap \bar{B}) = 0.5$

Find

a) $P(A)$ b) $P(A \cap B)$ c) $P(A \cup B)$

a) $P(A) = 1 - P(\bar{A}) = 1 - 0.3 = \boxed{0.7}$

b) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$0.5 = 0.7 - P(A \cap B)$

$P(A \cap B) = 0.7 - 0.5 = \boxed{0.2}$

c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 + 0.4 - 0.2 = \boxed{0.9}$

[7] Multiplication Rule: "Intersection of two independent events"

If A and B are independent events

$$P(A \cap B) = P(A)\,P(B)$$

لازم اتذكر انو شي ضرب، الدق حالة "independent events"

with replacement "Independent"

without replacement "Dependent"

Rolling a Die, Tossing a Coin

Ex (10):

Probability of getting number (6) when rolling a die twice.

$P(6 \text{ on roll (1) and } 6 \text{ on roll (2)}) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$

$= \boxed{\frac{1}{36}}$