Relation

$A = \{1,2\}$ $B = \{3,4\}$

$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

$\underbrace{\{(1,3), (1,4)\}}_{R} \subseteq A \times B$

① Reflexive $\forall x \in A \rightarrow (x,x) \in R$

$A = \{1,2,3\}$

$R = \{(1,1), (2,3)\}$

$(1,1) \in R$
$\wedge (2,2) \in R$
$\wedge (3,3) \in R$

not reflexive

$R_2 = \{(1,1), (2,2)\}$ X $(3,3) \notin R$

$R_3 = \{(1,1), (1,2), (2,2), (3,3)\}$ Ref

② Irreflexive $\forall x \in A \ (x,x) \notin R$

$A = \{1,2\}$

$R = \{(1,2), (2,2)\}$ X

$A = \{1,2\}$

$R = \{(1,2), (2,1)\}$ $(1,1) \notin R \wedge (2,2) \notin R$ Irref

Irrf

$R = \{(1,1), (2,2), (1,2)\}$ X

not Irr ref

$R = \{(1,1), (2,2)\}$ ref, not Irref

Symmetric

$\forall (a,b) \in R \rightarrow (b,a) \in R$

$A = \{1,2,3\}$

$R = \{(2,2), (3,1)\}$ not symm $\overset{T}{(3,1) \in R} \nrightarrow \overset{F}{(1,3) \in R}$

not ref not Irref

$R = \{(3,1), (1,3), (2,3)\}$ not symm $(2,3) \in R \nrightarrow (3,2) \in R$

$R = \{(2,2), (1,1)\}$ not ref, not Irell, symm

$R = \{(2,2), (3,1), (1,3)\}$ symm

Antisymmetric

$\forall (a,b) \in R \wedge (b,a) \in R \longrightarrow a = b \equiv a \neq b \longrightarrow (a,b) \notin R$ or $(b,a) \notin R$

$A = \{1,2\}$

$R = \{(1,2), (2,2), (2,1)\}$ Not Antisym

$(1,2) \wedge (2,1) \nrightarrow 1=2$

$R = \{(1,1), (2,2)\}$ Ref, not IRRF, Symm, Antisym

$R = \{(1,2), (2,2)\}$ not Symm, Antisym

Scanned with CamScanner

transitive

$A = \{1, 2, 3\}$

$\forall (a,b) \in R \wedge (b,c) \in R \longrightarrow (a,c) \in R \equiv (a,c) \notin R \longrightarrow \exists (a,b) \notin R$ or $(b,c) \notin R$

$R = \{(1,2), (2,1), (3,1)\}$

(a,b)	(b,c)	?? (a,c)
$(1,2)$	$(2,1)$	$(1,1) \notin R$

not transitive

$A = \{1, 2\}$

$R = \{ (1,2), (2,1), (1,1), (2,2) \}$

(a,b)	(b,c)	(a,c)
$(1,2)$	$(2,1)$	$(1,1) \in R$
$(1,2)$	$(2,2)$	$(1,2) \in R$
$(2,1)$	$(1,1)$	$(2,1) \in R$
$(2,1)$	$(1,2)$	$(2,2) \in R$
$(1,1)$	$(1,2)$	$(1,2) \in R$
$(2,2)$	$(2,1)$	$(2,1) \in R$

transitive

Scanned with CamScanner

$R = \{(1,2), (2,1), (1,1), (3,1)\}$

(a,b)	(b,c)	(a,c)
$(1,2)$	$(2,1)$	$(1,1) \in R$
$(2,1)$	$(1,1)$	$(2,1) \in R$
$(2,1)$	$(1,2)$	$(2,2) \notin R$

not transitive

$R = \{(1,2), (3,1), (3,2)\}$

(a,b)	(b,c)	(a,c)
$(3,1)$	$(1,2)$	$(3,2) \in R$

transitive

$(1,2)$ $(2, \ )$ $(1, \ )$

or

$R = \{(1,2), (3,2), (1,1)\}$

$$M_R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 \end{array}$$

Refl $\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$

$M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

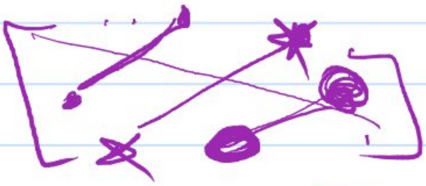
Irref $\begin{bmatrix} 0 & & \\ & 0 & \\ & & 0 \end{bmatrix}$

Ref

$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

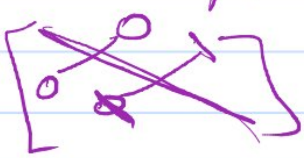
$\begin{bmatrix} 0 & 1 & 1 \\ & 0 & 1 \\ 1 & & 0 \end{bmatrix}$

Irref

Symm = 

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$M_R = M_R^T$



$$\begin{bmatrix} & 1 & 0 \\ 0 & & 0 \\ 0 & 1 & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$ Symm

Not AntiSymm

$$M_{R^2} \subseteq M_R$$

$$R_1 \cup R_2$$

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \wedge \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M_{R_1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad M_{R_2} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$M_{R_1 \odot R_2} = M_{R_1} \odot M_{R_2}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

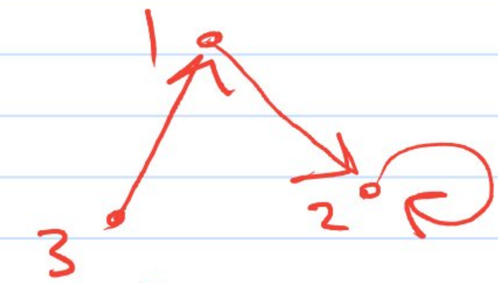
$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad \begin{matrix} 1 & 0 \\ \phi & 1 \\ \phi & 0 \end{matrix}$$

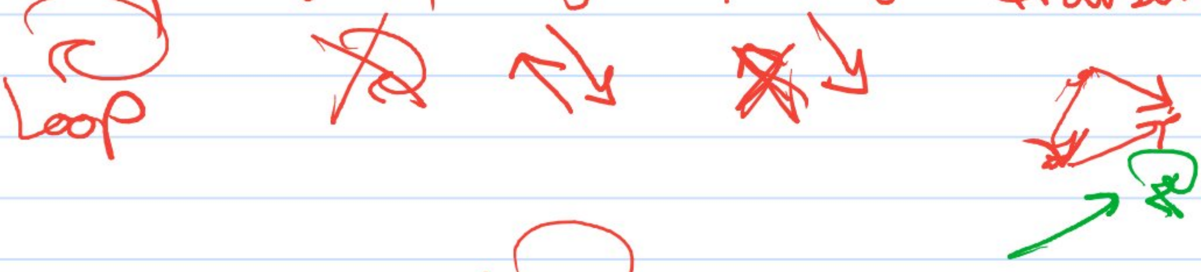
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \subseteq \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

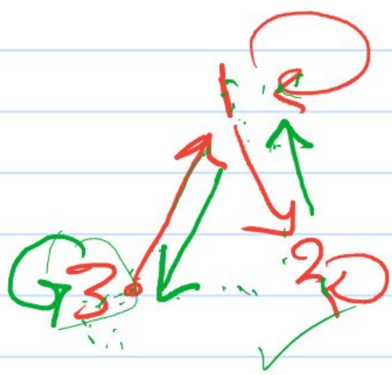
transit

$R = \{ (1,2)\ (3,1)\ (2,2) \}$

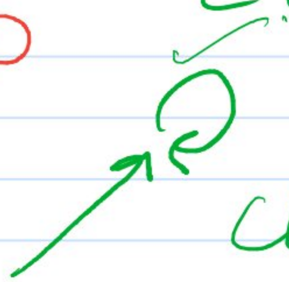


Ref Irref Sym Antisym Transitive





$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & \boxed{0} \end{bmatrix}$$



not Ref

not Irref

not Symm

AntiSymm

not Transitive

closure ref

$= R \cup \{ \quad \}$

closure sym

$= R \cup \{ \quad \}$