1. Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts.
b) Miami is the capital of Florida.
c) $2 + 3 = 5$.
d) $5 + 7 = 10$.
e) $x + 2 = 11$.
f) Answer this question.

a) Yes, T **b)** Yes, F **c)** Yes, T **d)** Yes, F **e)** No **f)** No

8. Let p, q, and r be the propositions

p : You have the flu.
q : You miss the final examination.
r : You pass the course.

Express each of these propositions as an English sentence.

a) $p \rightarrow q$
b) $\neg q \leftrightarrow r$
c) $q \rightarrow \neg r$
d) $p \vee q \vee r$
e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$
f) $(p \wedge q) \vee (\neg q \wedge r)$

(a) If you have the flu, you miss the final examination.
(b) You pass the course if and only if you take the final exam.
(c) If you miss the final examination, you will not pass the course.
(d) You either have the flu, miss the final examination, or pass the course
(e) If you have the flu, you will fail the course, or if you miss the final exam, you will fail the course.
(f) Either you have the flu and miss the final exam, or you don't miss the final exam and pass the course.

9. Let p and q be the propositions

p : You drive over 65 miles per hour.
q : You get a speeding ticket.

Write these propositions using p and q and logical connectives.

a) You do not drive over 65 miles per hour. — **a)** $\neg p$
b) You drive over 65 miles per hour, but you do not get a speeding ticket. — **b)** $p \wedge \neg q$
c) You will get a speeding ticket if you drive over 65 miles per hour. — **c)** $p \rightarrow q$
d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket. — **d)** $\neg p \rightarrow \neg q$
e) Driving over 65 miles per hour is sufficient for getting a speeding ticket. — **e)** $p \rightarrow q$
f) You get a speeding ticket, but you do not drive over 65 miles per hour. — **f)** $q \wedge \neg p$
g) Whenever you get a speeding ticket, you are driving over 65 miles per hour. — **g)** $q \rightarrow p$

7. Use De Morgan's laws to find the negation of each of the following statements.

a) Jan is rich and happy.
b) Carlos will bicycle or run tomorrow.
c) Mei walks or takes the bus to class.
d) Ibrahim is smart and hard working.

7. a) Jan is not rich, or Jan is not happy. **b)** Carlos will not bicycle tomorrow, and Carlos will not run tomorrow. **c)** Mei does not walk to class, and Mei does not take the bus to class. **d)** Ibrahim is not smart, or Ibrahim is not hard working.

9. Show that each of these conditional statements is a tautology by using truth tables.

a) $(p \wedge q) \rightarrow p$
c) $\neg p \rightarrow (p \rightarrow q)$
e) $\neg(p \rightarrow q) \rightarrow p$

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

12. Determine whether these biconditionals are true or false.

a) $2 + 2 = 4$ if and only if $1 + 1 = 2$.
b) $1 + 1 = 2$ if and only if $2 + 3 = 4$.
c) $1 + 1 = 3$ if and only if monkeys can fly.
d) $0 > 1$ if and only if $2 > 1$.

a) 2 + 2 = 4 if and only if 1 + 1 = 2 — **T <-> T is T**
b) 1 + 1 = 2 if and only if 2 + 3 = 4 — **T <-> F is F**
c) 1 + 1 = 3 if and only if monkey can fly — **F <-> F is T**
d) 0 > 1 if and only if 2 > 1 — **F <-> T is F**

26. How many rows appear in a truth table for each of these compound propositions?

a) $(q \rightarrow \neg p) \vee (\neg p \rightarrow \neg q)$ --> 4 rows
b) $(p \vee \neg t) \wedge (p \vee \neg s)$ --> 8 rows
c) $(p \rightarrow r) \vee (\neg s \rightarrow \neg t) \vee (\neg u \rightarrow v)$ --> 2^6 rows
d) $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$ --> 32 rows

27. Construct a truth table for each of these compound propositions.

a) $p \wedge \neg p$
c) $(p \vee \neg q) \rightarrow q$
b) $p \vee \neg p$

a)

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

c)

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \rightarrow q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

29. Construct a truth table for each of these compound propositions.

a) $(p \vee q) \rightarrow (p \oplus q)$
b) $(p \oplus q) \rightarrow (p \wedge q)$
c) $(p \vee q) \oplus (p \wedge q)$
d) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
e) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
f) $(p \oplus q) \rightarrow (p \oplus \neg q)$

For parts (a), (b), (c),

p	q	$(p \vee q) \rightarrow (p \oplus q)$	$(p \oplus q) \rightarrow (p \wedge q)$	$(p \vee q) \oplus (p \wedge q)$
T	T	F	T	F
T	F	T	F	T
F	T	T	F	T
F	F	T	T	F

16. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are equivalent. Use Truth Table

20. Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent. Use Truth Table

27. Show that $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are logically equivalent. Use Truth Table