Faculty of Computer & Information Sciences
Ain Shams University
Course Name: Discrete Math and LInear Algebra
Offering Dept: Scientific Computing
Academic year: 2015-2016
Instructors: Prof. Dr. Essam Khalefa & Dr. Mohammed Marey

Exam: (Final)11/1/2016
Year: (1^{st} term) 2^{nd} year
Duration: 3 hours
Total Grade: 95 (+ 5 bonus) points

كلية الحاسبات والمعلومات

Part 1: Discrete Mathematics (DM) - Solve$\left(\boldsymbol{DM1} \wedge (\boldsymbol{DM2} \vee \boldsymbol{DM3})\right)$

Questn DM1 — 25 points

9-Find the integer *a* such that $a \equiv -11 \ (\mathbf{mod}\ 21)$ and $80 \leq a \leq 100$.

10- Find each of these values.

a) $(-133 \ \mathbf{mod}\ 23 + 261 \ \mathbf{mod}\ 23) \ \mathbf{mod}\ 23$ **b)** $(992 \ \mathbf{mod}\ 32)3 \ \mathbf{mod}\ 15$

11- Find gcd(1000,5050) and lcm(1000,5050) using prime factorization method

12- Suppose that $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix}$ wzhere a, b, and c ∈ R, using mathematical induction show that $A^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{bmatrix}$ for every positive integer n.

13- Let P (n) be the statement that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps. The parts of this exercise outline a strong induction proof that P(n) is true for n ≥ 18.

Question DM2 — 25 points

1- Give the converse, the contrapositive, and the inverse of the conditional statement "If $n > 3$ then $n^2 > 9$"

2- Let P(x) ::="Student x knows LA" and let Q(y) ::= "Class y contains a student who knows calculus." Express each of these as quantifications of P(x) and Q(y).

a) Some students know LA.
b) Not every student knows LA.
c) Every class has a student in it who knows LA.
d) Every student in every class knows LA.
e) There is at least one class with no students who know LA.

3- Let P(m,n) ::="m dividesn," where the domain for both variables consists of all positive integers. Determine the truth values of each of these statements.

a) P(4, 5) **b)** P(2, 4) **c)** ∀m ∀n P(m,n) **d)** ∃m ∀n P(m,n) **e)** ∃n ∀m P(m,n) **f)** ∀n P(1,n)

4- Prove that if m + n and n + p are even integers, then m + p is even, where m, n, and p are integers.

5- Show that p ↔ q and (p ∧ q) ∨ (¬p ∧ ¬q) are logically equivalent by developing a series of logically equivalences.

6- Translate the following specifications into English where F(p) ::= "Printer p is out of service," B(p) ::="Printer p is busy," L(j) ::= "Print job j is lost," and Q(j) ::="Print job j is queued."

a) ∃p(F (p) ∧ B(p)) → ∃jL(j) **b)** ∀pB(p) → ∃jQ(j)
c) ∃j (Q(j) ∧ L(j)) → ∃pF (p) **d)** (∀pB(p) ∧ ∀jQ(j)) → ∃jL(j)

7- Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

a) $\forall x \forall y(x^2 = y2 \rightarrow x = y)$ **b)** $\forall x \exists y(y^2 = x)$ **c)** $\forall x \forall y(xy \geq x)$

8- Determine whether each of these conditional statements is true or false.

a) If cat can fly, then you can pass MATH 3 exam
a) If cat cannot fly, then you can fly
a) If you can fly, then the cat can fly
a) If you can fly, then 25+25+25+25=0

I. Use rules of inference to show that

$$u \rightarrow q,\ \ r \vee s,\ \ \neg s \rightarrow \neg p,\ \ ((\neg p) \wedge r) \rightarrow u,\ \ \neg s,\ \ \therefore\ q$$

Question DM3 **25 points**

14- Let R1 = {(a, b) | a divides b} and R2 = {(a, b) | a is a multiple of b}. be two relations on the set of all positive integers, respectively. Find $R1 \cup R2$, $R1 \cap R2$, $R1 - R2$, $R2 - R1$, and $R1 \oplus R2$.

15- Let $R1$ and $R2$ be relations on a set A represented by the matrices $M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, Find the matrices that represent $R1 \cup R2$, $R1 \cap R2$, $R2 \circ R1$, $R1 \circ R1$, $R1 \oplus R2$, R_1^{-1}, symmetric closure of R_1, reflexive closure of R_1, and transitive closure of R_1^{-1}.

16- Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation.

17 -Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $ad = bc$. Show that R is an equivalence relation.

18- Let m be an integer with m > 1. Show that the relation = {(a, b) | a ≡ b (mod m)} is an equivalence relation on the set of integers.

19- what are the equivalent classes of the following relations:

$R_1 = \{(a,b): a \equiv b\ (mod\ 7), a, b\ \in Z\}$,
$R_2 = \{(a,b): a \equiv b\ (mod\ 7), a, b\ \in even\}$,
$R_3 = \{(a,b): a \equiv b\ (mod\ 7), a, b\ \in Primes^+\}$,
$R_4 = \{(a,b): a \equiv b\ (mod\ 7), a, b\ \in \{1,2, \ldots ,14\}\}$,
$R_5 = \{(a,b): a \equiv b\ (mod\ 7), a, b\ \in \{1,2, \ldots ,5\}\}$

Part 2: Linear Algebra (LA) - Solve$(LA1 \wedge (LA2 \vee LA3))$

Question LA1 **25 points**

1- Find the parametric form of the general solution of the system whose augmented matrix is give by

$$A = \begin{bmatrix} 1 & 0 & -5 & 0 & -8 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2- Show that the transformation T defined by $T(x_1, x_2) = (x_1 - 2x_2\,, x_1 - 3, 2x_1 - 5x_2)$ is not a linear.

3- If v_1, v_2, v_3, and v_4 are in R^4 and $v_3 = 2v_1 + v_2$ then $\{v_1, v_2, v_3, v_4\}$ is a linearly independent set.

4- let $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, show that $\begin{bmatrix} n \\ k \end{bmatrix}$ is in span {u, v} for all real numbers n and k.

Question LA2 **25 points**

5- Let $A = \begin{bmatrix} -1 & -7 & -3 \\ 2 & 15 & 6 \\ 1 & 3 & 2 \end{bmatrix}$, find the third column of A^{-1} without computing the other columns.

6- Let $A = \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 5 \\ 1 & 5 \\ 3 & 4 \end{bmatrix}$, how to compute $A^{-1}B$ without computing A^{-1}.

[you do not have to find the solution, just write the idea as steps]

7- Given $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ where $A^2 = I$, use the partitions of the matrix to show that $M^2 = I$ where

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix}$$

8- Given $\mathbf{det}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 7$, find the determinants $\begin{bmatrix} 5g & 5h & 5i \\ 2d + a & 2e + b & 2f + c \\ a & b & c \end{bmatrix}$.

9- Let H be the set of points inside and on the unit circle in the xy-plane, i.e. $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 \leq 1 \right\}$, show that H is not a subspace of R^2.

10- Given subspace H & M of a vector space V, if $H + M = \{w : w = u + v;\ (u \in H) \land (v \in M)\}$, show that

1) H+M is a subspace of V.
2) H is a subspace of H+M.

Question LA3 **25 points**

11- Diagonalize the following matrix $A = \begin{bmatrix} 5 & -3 & 0 & 9 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$, then find A^{50} if possible.

[show the following: characteristic equation, eginvalues, bases of eigenspaces, matrix D, matrix P]

12- Given $p_1 = \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}$, $p_2 = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$, $p_3 = \begin{bmatrix} 3 \\ -4 \\ -7 \end{bmatrix}$ find the following: length of p_1 ; unit vector of p_2; distance between p_2 and p_1, finally determine whether the set $\{p_1, p_2, p_3, p_4\}$ is orthogonal set.

13-Find the matrix of quadratic form, assume $x \in R^5$ and $4x_1^2 + 4x_2^2 - 3x_3^2 + 2x_4^2 + 3x_1x_2 - 5x_3x_4 - 4x_1x_4$.

Good Luck