FCIS – Ain Shams University
Subject: Discrete Math
Exam: (Midterm) /12/2020
Year: (2nd) undergraduate

Examiner: Dr. ***Mohamed Abdel-Aal***
Offering Dept.: Scientific Computing
Academic year: 1st term 2020-2021
Duration: 45 minutes

Discrete Math --Model- B- **The total marks: 20**

Answer the following questions:

1. **[4 Marks]**

Prove that for every positive integer *n*:

$$1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$$

2. **[3 Marks]**

State the converse, inverse and contrapositive of the proposition:

'***If it rains today, then I will drive to work.'***

3. **[3 Marks]**

Determine the **truth value** of each of these five statements if the domain of all variables is the real numbers ***R*** (mention one counter example or one explanation for false statements):

1) $\forall x\ \exists y\ (x^2 = y)$ 2) $\forall x\ \exists y\ (x = y^2)$ 3) $\exists x\ \forall y\ (x.y = 0)$

4) $\exists x\ \exists y\ (x + y \neq y + x)$ 5) $\forall x\ \exists y\ (x / y = 1)$ 6) $\forall x\ \exists y\ (x . y = 1)$

4. **[3 Marks]**

Prove that if *m* and *n* are integers and *m.n* is even, then *m* is even or *n* is even.

5. **[3 Marks]**

Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent by **a truth table** and by **developing a series of logically equivalences**.

6. **[4 Marks]**

Show that these premises conclude $\neg \mathbf{q} \rightarrow s$ using rules of inference and logical equivalences if needed:

$$\mathbf{p \rightarrow q,\ \neg p \rightarrow r,\ and\ r \rightarrow s}$$

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Good Luck

*Dr. **Mohamed Abdel-Aal***

Rules of Inference:

Modus ponens: $(p \wedge (p \rightarrow q)) \rightarrow q$
Modus tollens: $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$
Hypothetical syllogism:
$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
Disjunctive syllogism $((p \vee q) \wedge \neg p) \rightarrow q$
Addition: $p \rightarrow (p \vee q)$
Simplification : $(p \wedge q) \rightarrow p$
Conjunction: $((p) \wedge (q)) \rightarrow (p \wedge q)$
Resolution: $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$

SOME Logical Equivalences:

$(p \vee q) \vee r \equiv p \vee (q \vee r)$
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$
$p \vee (p \wedge q) \equiv p$, $p \wedge (p \vee q) \equiv p$
$p \vee \neg p \equiv \mathbf{T}$, $p \wedge \neg p \equiv \mathbf{F}$
$p \rightarrow q \equiv \neg p \vee q$

$P \rightarrow q \equiv \neg q \rightarrow \neg p$
$p \vee q \equiv \neg p \rightarrow q$,
$p \wedge q \equiv \neg(p \rightarrow \neg q)$
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$