FCIS – Ain Shams University
Subject: Discrete Math
Exam: (Midterm) /12/2020
Year: (2nd) undergraduate

Examiner: Dr. ***Mohamed Abdel-Aal***
Offering Dept.: Scientific Computing
Academic year: 1st term 2020-2021
Duration: 45 minutes

Discrete Math. - Model -A- **The total marks: 20**

Answer the following questions:

1. **[4 Marks]**

Prove that for every positive integer n

$$\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots+\frac{1}{2^n}=1-\frac{1}{2^n}$$

2. **[3 Marks]**

Show whether the statement: $[\neg p \wedge (p \vee q)] \rightarrow q$ is a ***Tautology*** or a ***Contradiction*** by developing a series of **logical equivalences** and constructing a **truth table.**

3. **[3 Marks]**

State the converse, inverse and contrapositive of the proposition:

'***If Jack plays his guitar then Sara will sing'***.

4. **[3 Marks]**

Let P(m,n) ::="m divides n," where the domain for both variables consists of all positive integers. **Determine** the truth values of each of these statements.

a) P(4, 6) b) P(2, 4) c) ∀m ∀n P(m,n)

d) ∃m ∀n P(m,n) e) ∃n ∀m P(m,n) f) ∀n P(1,n)

5. **[3 Marks]**

Prove that if *n* is a positive integer, then *n* is odd **if and only if** 5n + 6 is odd.

6. **[4 Marks]**

Show that these premises concludes **t** using rules of inference and logical equivalences if needed:

$\neg p \wedge q$, $r \rightarrow p$, $\neg r \rightarrow s$, and $s \rightarrow t$.

Good Luck

Dr. ***Mohamed Abdel-Aal***

Rules of Inference:

Modus ponens: $(p \wedge (p \rightarrow q)) \rightarrow q$

Modus tollens: $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$

Hypothetical syllogism:

$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

Disjunctive syllogism $((p \vee q) \wedge \neg p) \rightarrow q$

Addition: $p \rightarrow (p \vee q)$

Simplification : $(p \wedge q) \rightarrow p$

Conjunction: $((p) \wedge (q)) \rightarrow (p \wedge q)$

Resolution: $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$

SOME Logical Equivalences:

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$p \vee (p \wedge q) \equiv p$, $p \wedge (p \vee q) \equiv p$

$p \vee \neg p \equiv \mathbf{T}$, $p \wedge \neg p \equiv \mathbf{F}$

$p \rightarrow q \equiv \neg p \vee q$

$P \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \vee q \equiv \neg p \rightarrow q$,

$p \wedge q \equiv \neg(p \rightarrow \neg q)$

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$