Statistical Analysis CIS240
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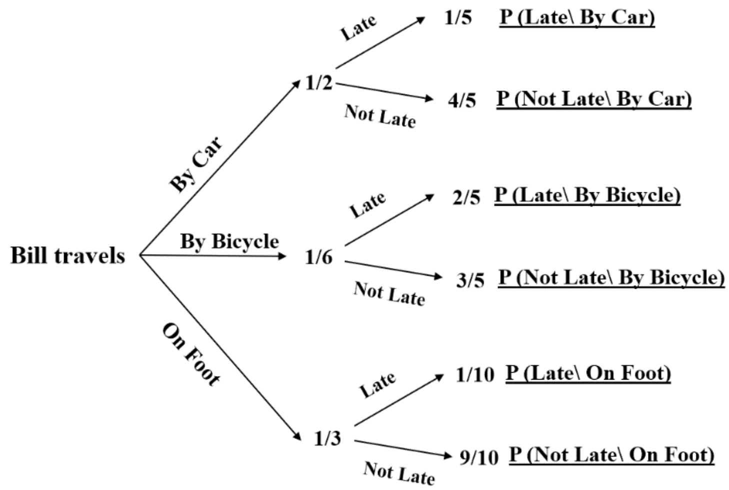
Conditional Probability and Bayes Rule Solved Problems

(1) On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $^1/_2$, $^1/_6$ and $^1/_3$ respectively. The probability of being late when using these methods of travel is $^1/_5$, $^2/_5$ and $^1/_{10}$ respectively.

a) Find the probability that on a randomly chosen day



i. Bill travels by foot and is late,

P(On Foot and Late) = (1/10) (1/3) = $\frac{1}{30}$ = **<u>0.0333</u>**

ii. Bill is not late.

P(Not Late) = *P(Not Late and By Car) + P(Not Late and By Bicycle) + P(Not Late and On Foot)*

P(Not Late) = *P(Not Late \ By Car) P(By Car) + P(Not Late \ By Bicycle) P(By Bicycle)+ P(Not Late \ On Foot) P(On Foot)*

P(Not Late) = (4/5) (1/2) + (3/5) (1/6) + (1/3) (9/10) = $\frac{4}{5}$ = **<u>0.8</u>**

b) Given that Bill is late, find the probability that he did not travel on foot.

P(Late) = 1- P(Not Late) = 1 - 0.8 = **<u>0.2</u>**

P(On Foot | Late) = $\frac{P(Late\ \ On\ Foot)P(On\ Foot)}{P(Late)}$ =

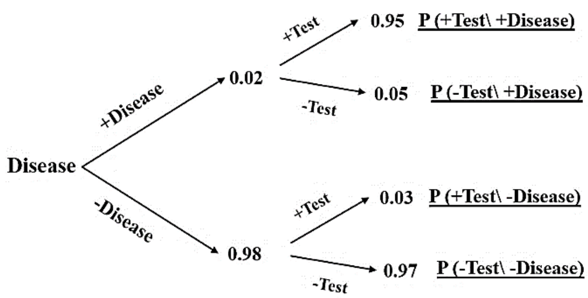
$$\frac{\left(\frac{1}{10}\right)\left(\frac{1}{3}\right)}{0.2} = \frac{\frac{1}{30}}{0.2} = \frac{1}{6} = \underline{\mathbf{0.167}}$$

P(Not On Foot | Late) = 1 - P(On Foot | Late) =

$$= 1 - 0.167 = \frac{1}{6} = \underline{0.833}$$

(2) A disease is known to be present in 2% of a population. A test is developed to help determine whether or not someone has the disease. Given that a person has the disease, the test is positive with probability 0.95. Given that a person does not have the disease, the test is positive with probability 0.03.

i. A person is selected at random from the population and tested for this disease. Find the probability that the test is positive.



P (+Test) *= P (+Test ∩ +Disease) + P (+Test ∩ -Disease)*

P (+Test) *= P (+Test\ +Disease) P (+Disease) + P (+Test\ -Disease) P (-Disease)*

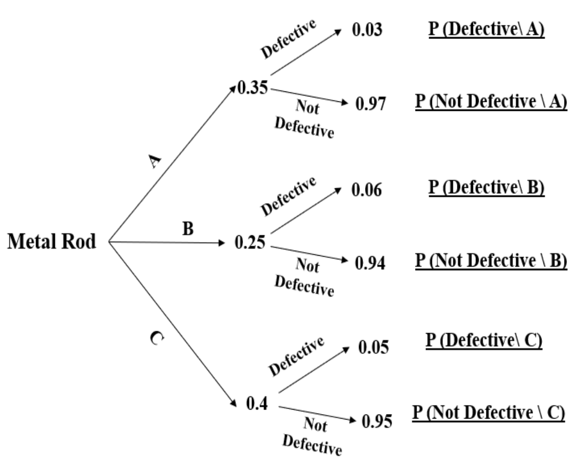
P (+Test) *= (0.95) (0.02) + (0.03) (0.98) =* ***<u>0.0484</u>***

ii. A doctor randomly selects a person from the population and tests him for the disease. Given that the test is positive, find the probability that he does not have the disease.

P (-Disease\ +Test) $= \frac{P\,(+Test \backslash -Disease)\, P\,(-Disease)}{P\,(+Test)} = \frac{(0.03)\,(0.98)}{0.0484} =$ ***<u>0.607</u>***

(a) In a factory, machines A, B and C are all producing metal rods of the same length. Machine A produces 35% of the rods, machine B produces 25% and the rest are produced by machine C. Of their production of rods, machines A, B and C produce 3%, 6% and 5% defective rods respectively.

i. Find the probability that a randomly selected rod will be defective.



P (Defective) = P (Defective ∩ A) + P (Defective ∩ B) + P (Defective ∩ C)

P (Defective) = P (Defective \ A) P(A) +P(Defective\B)P(B)+P(Defective\ C) P(C)

P (Defective) = (0.03) (0.35) + (0.06) (0.25) + (0.05) (0.4) = **<u>0.0455</u>**

ii. Given that a randomly selected rod, find the probability that it is produced by machine A.

P(A \ Defective) $= \frac{P\,(Defective \backslash A)\, P(A)}{P\,(Defective)} = \frac{(0.03)\,(0.35)}{0.0455} =$ **<u>0.231</u>**