

CHW 261: Logic Design

Tutorial Sheet 1

1) List numbers from 8 to 28 in base 12.

Base 12: 0 → 11: 0,1,2,3,4,5,6,7,8,9, A, B

$$12^1 \quad 12^0$$

$$12 \quad 1$$

Dec	Base12
8	0 8
9	0 9
10	0 A
11	0 B
12	1 0
13	1 1
14	1 2
15	1 3
16	1 4
17	1 5
18	1 6
19	1 7
20	1 8
21	1 9
22	1 A
23	1 B
24	2 0
25	2 1
26	2 2
27	2 3
28	2 4

2) Largest binary number with 16 bits =

$$2^{15}2^{14}2^{13}2^{12} \quad 2^{11}2^{10}2^92^8 \quad 2^72^62^52^4 \quad 2^32^22^12^0$$

Bin: 1111 1111 1111 1111

Dec: $2^0+2^1+2^2+\dots+2^{15}= 65535$

Hex: F F F F

3) How many bits are needed to represent 205?

N: number of bits

$$N \rightarrow 2^N : 0 \rightarrow 2^N - 1$$

$$N = 2 \rightarrow 0 : 2^2 - 1 = 0 : 3$$

$$N = 5 \rightarrow 0 : 2^5 - 1 = 0 : 31$$

$$N = 7 \rightarrow 0 : 2^7 - 1 = 0 : 127$$

$$N = 8 \rightarrow 0 : 2^8 - 1 = 0 : 255$$

We need 8 bits to be able to represent the number 205.

4)

$$\text{a) } N = 7 \text{ bits} \rightarrow 0 : 2^7 - 1 = 127$$

b) Largest number that can be represented by N digits in Hexadecimal = $16^N - 1$

$$N = 3 \rightarrow 0 : 16^3 - 1 = 0 : 4095$$

5) Convert to Decimal:

a. $(10110.0101)_2$

$$\text{Dec} = 2^1 + 2^2 + 2^4 + 2^{-2} + 2^{-4} = 22.3125$$

b. $(121)_3$

$$\text{Dec} = 1 * 3^0 + 2 * 3^1 + 1 * 3^2 = 16$$

c. $(345)_6$

$$\text{Dec} = 5 * 6^0 + 4 * 6^1 + 3 * 6^2 = 137$$

d. $(77.7)_8$

$$\text{Dec} = 7*8^0 + 7*8^1 + 7*8^{-1} = 63.875$$

e. $(435)_8$

$$\text{Dec} = 5 * 8^0 + 3*8^1 + 4*8^2 = 285$$

f. $(198)_{12}$

$$\text{Dec} = 8 * 12^0 + 9 * 12^1 + 1*12^2 = 260$$

g. $(AC5)_{16}$

$$\text{Dec} = 5* 16^0 + 12*16^1 + 10*16^2 = 2757$$

h. $(16.5)_{16}$

$$\text{Dec} = 6 * 16^0 + 1*16^1 + 5*16^{-1} = 22.3125$$

6)

a. $(28.125)_{10}$ to binary

$$\text{Bin} = (11110 . 001)_2$$

$$28/2 = 14 \rightarrow 0$$

$$14/2 = 7 \rightarrow 1$$

$$7/2 = 3 \rightarrow 1$$

$$3/2 = 1 \rightarrow 1$$

$$1/2 = 0 \rightarrow 1$$

$$0.125 * 2 = 0.25$$

$$0.25 * 2 = 0.5$$

$$0.5 * 2 = 1.0$$

b. $(157.128)_{10}$ to hexadecimal

Hex = $(9D.20C)_{16}$

$$157/16 = 9 \rightarrow 13$$

$$9/16 = 0 \rightarrow 9$$

$$.128 * 16 = 2.048$$

$$0.048 * 16 = 0.768$$

$$0.768 * 16 = 12.228$$

c. $(67.45)_{10}$ to octal

Oct = $(103.346)_8$

$$67/8 = 8 \rightarrow 3$$

$$8/8 = 1 \rightarrow 0$$

$$1/8 = 0 \rightarrow 1$$

$$0.45 * 8 = 3.6$$

$$0.6 * 8 = 4.8$$

$$0.8 * 8 = 6.4$$

d. $(2AC5)_{16}$ to octal (without converting to decimal)

$$2^3 2^2 2^1 2^0$$

$$8 \ 4 \ 2 \ 1$$

Hex	Bin
2	0 0 10
A	1 0 10
C	1 1 0 0
5	0 1 0 1

$$(2AC5)_{16} = (\ 0010 \ 1010 \ 1100 \ 0101 \)_2$$

$$2^2 2^1 2^0$$

$$4 \ 2 \ 1$$

Bin	Octal
101	5
00 0	0
0 11	3
101	5
010	2

$$(\ 0010 \ 1010 \ 1100 \ 0101 \)_2 = (\ 25305 \)_8$$

7) Perform the addition:

a) $(110110)_2 + (110101)_2$

110110

+

110101

1101011

b) $(15F)_{16} + (A7)_{16}$

1 5 F

+

A 7

206

$(22)_{10} = (16)_{16}$

$(16)_{10} = (10)_{16}$

c) $(35)_8 + (73)_8$

35

+

$$\begin{array}{r} 73 \\ \hline 130 \end{array}$$

$$(8)_{10} = (10)_8$$

$$(11)_{10} = (13)_8$$

8) Perform the multiplication:

a) $(367)_8 * (20)_8$

$$\begin{array}{r} 367 \\ * \\ 20 \\ \hline 000 \\ 756 \\ \hline 7560 \end{array}$$

$$(14)_{10} = (16)_8$$

$$(13)_{10} = (15)_8$$

$$b) (b73)_{16} * (15)_{16}$$

b73

*

15

—

39 3 F

B73

—

F06F

$$(35)_{10} = (23)_{16}$$

$$(57)_{10} = (39)_{16}$$

$$(16)_{10} = (10)_{16}$$