CH.2 Basic Structures: Sets, Functions, Sequences, Sums

Sets

- Introduction
- Power Sets
- Truth Sets and Quantifiers
- Set Operations
- Set Identities
- Computer Representation of Sets

❖ Sets

Sets are used to group objects together. Often, but not always, the objects in a set have similar properties.

For instance,

- all the students currently taking a course in discrete mathematics at any school

DEFINITION

A set is an unordered collection of objects, called elements or members of the set. A set is said to contain its elements. We write $a \in A$ to denote that a is an element of the set A. The notation $a \notin A$ denotes that a is not an element of the set A.

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It is common for sets to be denoted using uppercase letters. Lowercase letters are usually used to denote elements of sets.

EXAMPLES

1. The set O of odd positive integers less than 10 can be expressed by

$$O = \{1, 3, 5, 7, 9\}.$$

2. Nothing prevents a set from having seemingly unrelated elements.

{a, 2, Fred, New Jersey}

3. The set of positive integers less than 100 can be denoted by

{1, 2, 3,...,99}.

Another way to describe a set is to use. **set builder** notation

We characterize all those elements in the set by stating the property or properties they must have to be members

For instance,

4. the set O of all odd positive integers less than 10 can be written as

$O = \{x \mid x \text{ is an odd positive integer less than } 10\},$

or, specifying the universe as the set of positive integers, as

$O = \{x \in Z^+ \mid x \text{ is odd and } x < 10\}.$

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5. the set Q^+ of all positive rational numbers can be written as

$$\mathbf{Q}^+ = \{x \in \mathbf{R} \mid x = \frac{p}{q}, \text{ for some positive integers } p \text{ and } q\}.$$

6. Q ={p/q | p ∈ Z, q∈ Z, and q ≠ 0}, the set of rational numbers

7. The set {N, Z, Q, R} is a set containing four elements, each of which is a set.

DEFINITION

Two sets are equal if and only if they have the same elements. Therefore, if A and B are sets, then A and B are equal if and only if

$$\forall x(x \in A \leftrightarrow x \in B).$$

We write A = B if A and B are equal sets.

The sets {1, 3, 5} , {1, 3, 3, 3, 5, 5, 5, 5} and {3, 5, 1} are equal,

- **Note that** the order in which the elements of a set are listed does not matter. Note also that it does not matter if an element of a set is listed more than once,

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THE EMPTY SET is a special set that has no elements, and is denoted by Ø.

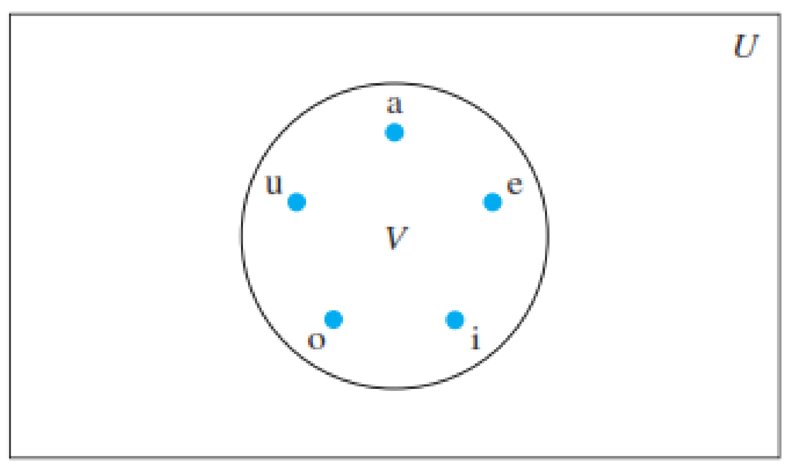
A set with one element is called a **singleton set**.

Venn Diagrams

In Venn diagrams the universal set U, which contains all the objects under consideration, is represented by a rectangle.

EXAMPLES

Draw a Venn diagram that represents V, the set of vowels in the English alphabet.



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Quiz

1. List the members of these sets.

a) $\{x \mid x \text{ is a real number such that } x^2 = 1\}$
b) $\{x \mid x \text{ is a positive integer less than } 12\}$
c) $\{x \mid x \text{ is the square of an integer and } x < 100\}$
d) $\{x \mid x \text{ is an integer such that } x^2 = 2\}$

2. Use set builder notation to give a description of each of these sets.

a) $\{0, 3, 6, 9, 12\}$
b) $\{-3, -2, -1, 0, 1, 2, 3\}$

Determine whether these statements are true or false.

a) $\emptyset \in \{\emptyset\}$
b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
c) $\{\emptyset\} \in \{\emptyset\}$
d) $\{\emptyset\} \in \{\{\emptyset\}\}$

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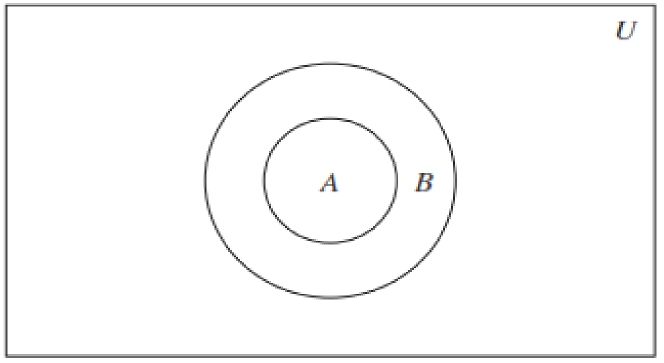
Subsets

The set A is a ***subset*** *of B if and only if every element of A is also an element of B. We use* the notation $A \subseteq B$ to indicate that A is a subset of the set B.



We see that $A \subseteq B$ if and only if the quantification $\forall x(x \in A \rightarrow x \in B)$ is true.

- **Note that** to show that A is not a subset of B we need only find one element $x \in A$ with $x \notin B$. $A \not\subseteq B$.
- To show that $A \subseteq B$, show that if x belongs to A then x also belongs to B.
- To show that $A \not\subseteq B$, find a single $x \in A$ such that $x \notin B$.

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THEOREM

For every set S,

(i) $\emptyset \subseteq S$ and (ii) $S \subseteq S$.

Proof

To show that $\emptyset \subseteq S$, we must show that $\forall x(x \in \emptyset \rightarrow x \in S)$ is true.

Because the empty set contains no elements, it follows that $x \in \emptyset$ is always false. It follows that the conditional statement $x \in \emptyset \rightarrow x \in S$ is always true,

because its hypothesis is always false and a conditional statement with a false hypothesis is true. (**<u>vacuous proof</u>**)

the proof of (ii) as an exercise.

When we wish to emphasize that a set A is a subset of a set B but that $A \neq B$, we write $A \subset B$ and say that A is a **proper subset of B.**

$$\forall x\,(x \in A \rightarrow x \in B) \wedge \exists x\,(x \in B \wedge x \notin A) \text{ is true.}$$

A = B if and only if $\forall x(x \in A \rightarrow x \in B)$ and $\forall x(x \in B \rightarrow x \in A)$ or equivalently if and only if $\forall x(x \in A \leftrightarrow x \in B)$, which is what it means for the A and B to be equal.

- *Showing Two Sets are Equal To show that two sets A and B are equal, show that* $A \subseteq B$ and $B \subseteq A$.

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$A = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$ and $B = \{x \mid x \text{ is a subset of the set } \{a, b\}\}$.
Note that these two sets are equal, that is, $A = B$.
Also note that $\{a\} \in A$, but $a \notin A$.

DEFINITION

Let S be a set. If there are exactly *n* distinct elements in S where *n* is a nonnegative integer, we say that S is a *finite set and that n is the* ***cardinality*** *of S. The cardinality of S is denoted* by |S|.

EXAMPLES

- Let A be the set of odd positive integers less than 10. Then |A|=5.
- Because the null set has no elements, it follows that |Ø| = 0.
- Let S be the set of letters in the English alphabet. Then |S|=26.

A set is said to be **infinite** if it is not finite.

- The set of positive integers is infinite.

Quiz

Determine whether each of these statements is true or false.

a) $x \in \{x\}$ **b)** $\{x\} \subseteq \{x\}$ **c)** $\{x\} \in \{x\}$

d) $\{x\} \in \{\{x\}\}$ **e)** $\emptyset \subseteq \{x\}$ **f)** $\emptyset \in \{x\}$

Determine whether these statements are true or false.

a) $\emptyset \in \{\emptyset\}$ **b)** $\emptyset \in \{\emptyset, \{\emptyset\}\}$

c) $\{\emptyset\} \in \{\emptyset\}$ **d)** $\{\emptyset\} \in \{\{\emptyset\}\}$

e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ **f)** $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

What is the cardinality of each of these sets?

a) $\{a\}$ **b)** $\{\{a\}\}$

c) $\{a, \{a\}\}$ **d)** $\{a, \{a\}, \{a, \{a\}\}\}$

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Power Sets

Given a set S, ***the power*** *set of S is the set of all subsets of the set S. The power set of S is* denoted by P(S).

EXAMPLES

- What is the power set of the set {0, 1, 2}?

P({0, 1, 2}) ={Ø, {0}, {1}, {2}, {0, 1}, {0, 2}, {1, 2}, {0, 1, 2}}.

- What is the power set of the empty set? What is the power set of the set {Ø}?

The empty set has exactly one subset, namely, itself. Consequently,

P(Ø) = {Ø}.

The set {Ø} has exactly two subsets, namely, Ø and the set {Ø} itself. Therefore, P({Ø}) ={Ø, {Ø}}.

❑ **Note that** If a set has *n* elements, then its power set has 2^n.

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Quiz Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ if and only if $A \subseteq B$.

EXAMPLE

Let $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. Determine whether each of the following is true or false and give a brief justification.

(i) $B \in \mathscr{P}(A)$
(ii) $B \in A$
(iii) $A \in \mathscr{P}(A)$
(iv) $A \subseteq \mathscr{P}(A)$
(v) $B \subseteq \mathscr{P}(A)$
(vi) $\{\{1\}, B\} \subseteq \mathscr{P}(A)$
(vii) $\varnothing \in \mathscr{P}(A)$
(viii) $\varnothing \subseteq \mathscr{P}(A)$.

Solution

(i) True: B is a subset of A so B is an element of its power set.

(ii) False: B is a set but the elements of A are numbers, so B is not an element of A.

(iii) True: since $A \subseteq A$ it follows that $A \in \mathscr{P}(A)$. In fact, as noted above, this is the case for any set A.

(iv) False: the elements of A are numbers whereas the elements of $\mathscr{P}(A)$ are sets (namely subsets of A). Hence the elements of A cannot also be elements of $\mathscr{P}(A)$, so $A \nsubseteq \mathscr{P}(A)$.

(v) False: for the same reasons as given in part (iv).

(vi) True: $\{1\} \in \mathscr{P}(A)$ (since $\{1\} \subseteq A$) and $B \in \mathscr{P}(A)$ (part (i)) so each element of the set $\{\{1\}, B\}$ is also an element of $\mathscr{P}(A)$; hence $\{\{1\}, B\} \subseteq \mathscr{P}(A)$.

(vii) True: since $\varnothing \subseteq A$, we have $\varnothing \in \mathscr{P}(A)$.

(viii) True: $\varnothing \subseteq X$ for every set X and $\mathscr{P}(A)$ is certainly a set, so $\varnothing \subseteq \mathscr{P}(A)$.

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Cartesian Products

Let A and B be sets. **The *Cartesian product*** *of A and B, denoted by* $A \times B$, *is the set of all* ordered pairs (a, b), where $a \in A$ and $b \in B$. Hence,

$$A \times B = \{(a, b) \mid a \in A \land b \in B\}.$$

EXAMPLES

What is the Cartesian product of $A = \{1, 2\}$ and $B = \{a, b, c\}$?

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2,c)\}.$$

- We use the notation A^2 to denote $A \times A$, the Cartesian product of the set A with itself.

The Cartesian product of more than two sets can also be defined.

The *Cartesian product* of the sets $A_1, A_2, \ldots, A_n$, denoted by $A_1 \times A_2 \times \cdots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, \ldots, a_n)$, where a_i belongs to A_i for $i = 1, 2, \ldots, n$. In other words,

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, \ldots, n\}.$$

EXAMPLE

What is the Cartesian product A × B × C, where A ={0, 1}, B ={1, 2}, and C ={0, 1, 2} ?

A × B × C ={(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2)}.

Using Set Notation with Quantifiers

What do the statements ∀x ∈ **R** $(x^2 \geq 0)$ and ∃x ∈ **Z** $(x^2 = 1)$ mean?

The statement ∀x ∈ **R** $(x^2 \geq 0)$ states that for every real number *x*, $x^2 \geq 0$. This statement can be expressed as "The square of every real number is nonnegative." This is a true statement.

The statement ∃x ∈ **Z** $(x^2 = 1)$ states that there exists an integer *x* such that $x^2 = 1$. This statement can be expressed as "There is an integer whose square is 1." This is also a true statement because x = 1 is such an integer (as is -1).

Truth Sets and Quantifiers

We will now tie together concepts from set theory and from predicate logic. Given a predicate P, and a domain D, we define the **truth set of *P*** **to be the set of elements *x* in *D* for which** P(*x*) is true. The truth set of P(x) is denoted by

$$\{x \in D \mid P(x)\}.$$

EXAMPLES

What are the truth sets of the predicates:

P(x) is " |x|=1" , *Q*(x) is " $x^2 = 2$," and *R*(x) is " |x|=x."

where the domain is the set of integers

- The truth set of P, {x ∈ Z | |x|=1}, is the set {-1, 1}.
- The truth set of Q, {x ∈ Z | $x^2 = 2$}. This is the empty ∅ Type equation here.set because there are no integers *x* for which $x^2 = 2$.

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- The truth set of R, $\{x \in Z \mid |x|=x\}$, is the set of integers for which $|x|=x$

Because $|x|=x$ if and only if $x \geq 0$, it follows that the truth set of R is N, the set of nonnegative integers.

Note that :

$\forall x P(x)$ is true over the domain U if and only if the truth set of P is the set U.

Likewise,

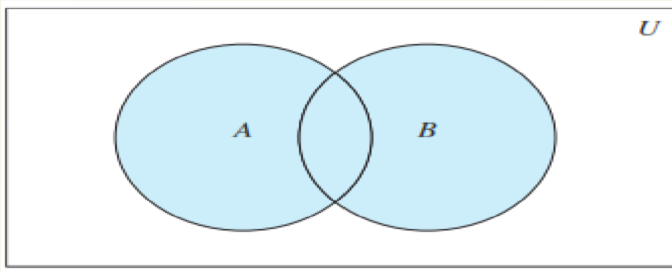
$\exists x P(x)$ is true over the domain U if and only if the truth set of P is nonempty.

❖ Set Operations

DEFINITIONS

- Let A and B be sets. The *union of the sets A and B*, *denoted by*
- $A \cup B$, is *the set that contains those elements that are either in A or in B, or in both*.

$$A \cup B = \{x \mid x \in A \lor x \in B\}.$$



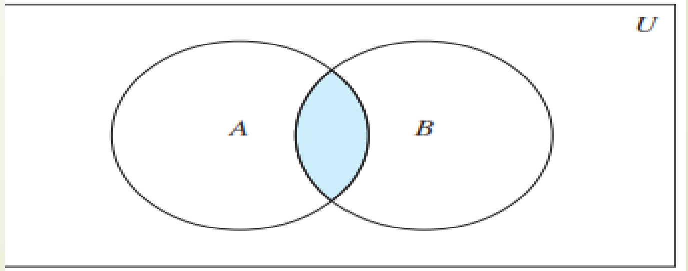
A ∪ B is shaded.

- The union of the sets {1, 3, 5} and {1, 2, 3} is the set {1, 2, 3, 5}; that is,

$\{1, 3, 5\} \cup \{1, 2, 3\} = \{1, 2, 3, 5\}.$

- Let A and B be sets. *The intersection of the sets* A and B, denoted by $A \cap B$, is the set containing those elements in both A and B.

$$A \cap B = \{x \mid x \in A \land x \in B\}.$$



A ∩ B is shaded.

- Two sets are called *disjoint if their intersection is the empty set.*

- $|A \cup B| = |A| + |B| - |A \cap B|.$

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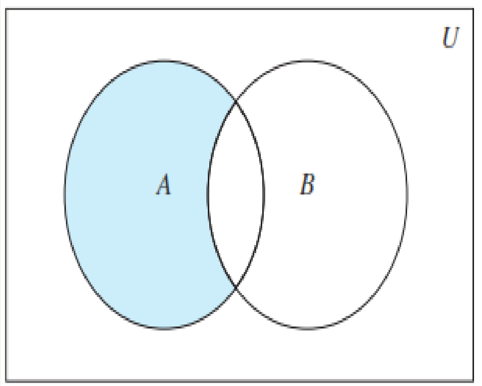
Difference

- Let A and B be sets. The difference of A and B, denoted by A - B, is the set containing those elements that are in A but not in B. The difference of A and B is also called the complement of B with respect to A.



A − B is shaded.

A-B: the set containing those elements in A but not in B

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

- A={1,3,5},B={1,2,3}, A-B={5}

complement

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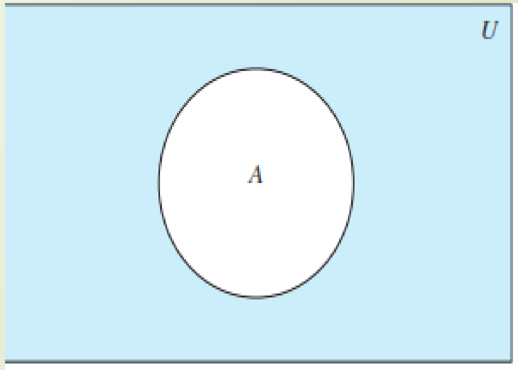
Once the universal set U is specified, the complement of a set can be defined **Complement** of A:



$$\overline{A} = \{x \mid x \notin A\}, \overline{A} = U - A$$

A-B is also called the complement of B with respect to A

EXAMPLES

- Let A be the set of positive integers greater than 10 (with universal set the set of all positive integers). Then

$$\overline{A} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}.$$

- The difference of {1, 3, 5} and {1, 2, 3} is the set {5}; that is, {1, 3, 5}-{1, 2, 3}={5}. This is different from the difference of {1, 2, 3} and {1, 3, 5}, which is the set {2}.

- The intersection of the sets {1, 3, 5} and {1, 2, 3} is the set {1, 3}; that is, {1, 3, 5} ∩{1, 2, 3}={1, 3}.

Quiz (1)

Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

a) $A \cup B$. **b)** $A \cap B$.
c) $A - B$. **d)** $B - A$.

What can you say about the sets A and B if we know that

a) $A \cup B = A$? **b)** $A \cap B = A$?
c) $A - B = A$? **d)** $A \cap B = B \cap A$?
e) $A - B = B - A$?

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Set Identities

We will prove several of these identities here, using three different methods.

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

- One way to show that two sets are equal is to show that each is a subset of the other
- membership tables
- Additional set identities can be established using those that we have already proved.

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EXAMPLE

Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$

we wand show that $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$ and $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$.

First, we will show that $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$.
We do this by showing that if *x* is in $\overline{A \cap B}$, then it must also be in $\overline{A} \cup \overline{B}$. Now suppose that x ∈ $\overline{A \cap B}$. By the definition of complement, x ∉ A ∩B. Using the definition of intersection, we see that the proposition ¬((x ∈ A) ∧ (x ∈ B)) is true.

By applying De Morgan's law for propositions, we see that ¬(x ∈ A) or ¬(x ∈ B). Using the definition of negation of propositions, we have x ∉ A or x ∉ B. Using the definition of the complement of a set, we see that this implies that x ∈ $\overline{A}$ or x ∈ $\overline{B}$. Consequently, by the definition of union, we see that x ∈ $\overline{A} \cup \overline{B}$. We have now shown that $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$.

Next, we will show that $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$. We do this by showing that if *x* is in $\overline{A} \cup \overline{B}$, then must also be in $\overline{A \cap B}$. Now suppose that *x* ∈ $\overline{A} \cup \overline{B}$. By the definition of union, we know that x ∈ $\overline{A}$ or x ∈ $\overline{B}$. Using the definition of complement, we see that x ∉ A or x ∉ B. Consequently, proposition ¬(x ∈ A) ∨ ¬(x ∈ B) is true.

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By De Morgan's law for propositions, we conclude that $\neg((x \in A) \land (x \in B))$ is true. By the definition of intersection, it follows that $\neg(x \in A \cap B)$. We now use the definition of complement to conclude that $x \in \overline{A \cap B}$. This shows that $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$

using set builder notation,

Solution: We can prove this identity with the following steps.

$\overline{A \cap B}$	$= \{x \mid x \notin A \cap B\}$	by definition of complement
	$= \{x \mid \neg(x \in (A \cap B))\}$	by definition of does not belong symbol
	$= \{x \mid \neg(x \in A \land x \in B)\}$	by definition of intersection
	$= \{x \mid \neg(x \in A) \lor \neg(x \in B)\}$	by the first De Morgan law for logical equivalences
	$= \{x \mid x \notin A \lor x \notin B\}$	by definition of does not belong symbol
	$= \{x \mid x \in \overline{A} \lor x \in \overline{B}\}$	by definition of complement
	$= \{x \mid x \in \overline{A} \cup \overline{B}\}$	by definition of union
	$= \overline{A} \cup \overline{B}$	by meaning of set builder notation

EXAMPLE

- Prove $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$
- (→): Suppose that $x\in A\cap(B\cup C)$ then $x\in A$ and $x\in B\cup C$. By definition of union, it follows that $x\in A$, and ($x\in B$ or $x\in C$). Consequently, $x\in A$ and $x\in B$ or $x\in A$ and $x\in C$
- By definition of intersection, it follows $x\in A\cap B$ or $x\in A\cap C$
- By definition of union, $x\in(A\cap B)\cup(A\cap C)$

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- (←): Suppose that $x \in (A \cap B) \cup (A \cap C)$
- By definition of union, $x \in A \cap B$ or $x \in A \cap C$
- By definition of intersection, $x \in A$ and $x \in B$, or $x \in A$ and $x \in C$
- From this, we see $x \in A$, and $x \in B$ or $x \in C$
- By definition of union, $x \in A$ and $x \in B \cup C$
- By definition of intersection, $x \in A \cap (B \cup C)$

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Set Identities

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TABLE 1 Set Identities.

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

EXAMPLE

Let A, B, and C be sets. Show that

$$\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}.$$

Solution: We have

$$\begin{aligned}
\overline{A \cup (B \cap C)} &= \overline{A} \cap (\overline{B \cap C}) && \text{by the first De Morgan law} \\
&= \overline{A} \cap (\overline{B} \cup \overline{C}) && \text{by the second De Morgan law} \\
&= (\overline{B} \cup \overline{C}) \cap \overline{A} && \text{by the commutative law for intersections} \\
&= (\overline{C} \cup \overline{B}) \cap \overline{A} && \text{by the commutative law for unions.}
\end{aligned}$$

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Set identities can also be proved using **membership tables. We consider each combination** of sets that an element can belong to and verify that elements in the same combinations of sets belong to both the sets in the identity. To indicate that an element is in a set, a 1 is used; to indicate that an element is not in a set, a 0 is used. (The reader should note the similarity between membership tables and truth tables.)

TABLE 2 A Membership Table for the Distributive Property.

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

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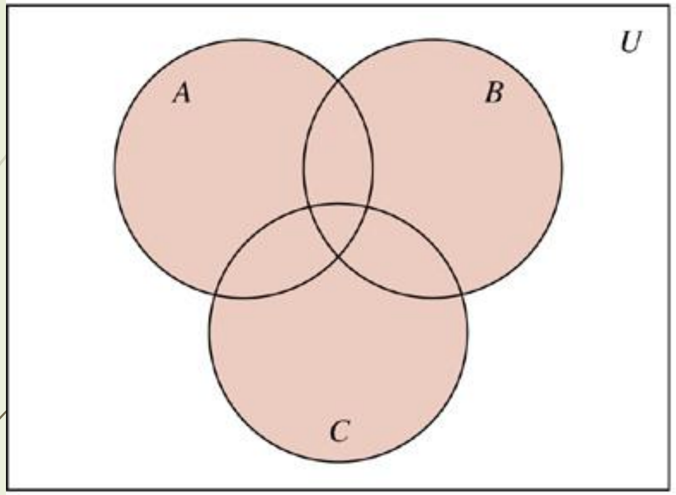
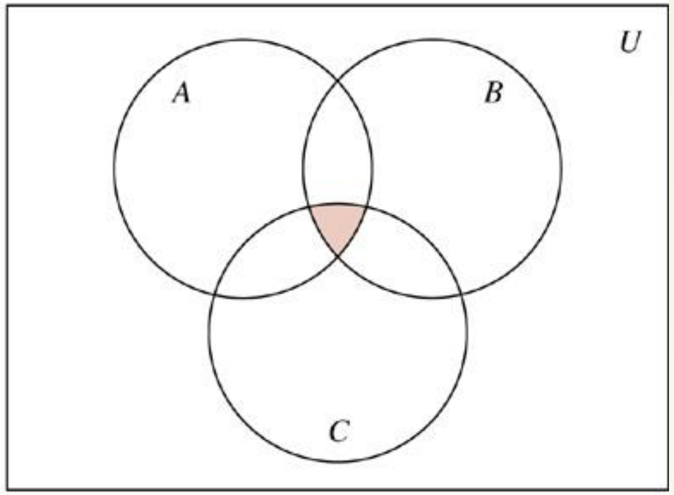
Because the columns for $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$ are the same, the identity is valid.

Generalized union and intersection



(a) $A \cup B \cup C$ is shaded.



(b) $A \cap B \cap C$ is shaded.

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A={0,2,4,6,8}, B={0,1,2,3,4}, C={0,3,6,9}
A⋃B⋃C={0,1,2,3,4,6,8,9}
A⋂B⋂C={0}

General case

- Union: $A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$
- Intersection $A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$
- Union: $A_1 \cup A_2 \cup \cdots \cup A_n \cup \cdots = \bigcup_{i=1}^{\infty} A_i$
- Intersection: $A_1 \cap A_2 \cap \cdots \cap A_n \cap \cdots = \bigcap_{i=1}^{\infty} A_i$
- Suppose A_i={1, 2, 3,..., i} for i=1,2,3,...

$$\bigcup_{i=1}^{\infty} A_i = \bigcup_{i=1}^{\infty} \{1,2,3,\ldots,i\} = \{1,2,3,\ldots\} = Z^+$$

$$\bigcap_{i=1}^{\infty} A_i = \bigcap_{i=1}^{\infty} \{1,2,3,\ldots,i\} = \{1\}$$

Lecture 4

EXAMPLE

Let A ={0, 2, 4, 6, 8}, B ={0, 1, 2, 3, 4}, and C ={0, 3, 6, 9}. What are A ∪ B ∪ C and A ∩ B ∩ C ?

The set A ∪ B ∪ C contains those elements in at least one of A, B, and C. Hence,

A ∪ B ∪ C ={0, 1, 2, 3, 4, 6, 8, 9}.

The set A ∩ B ∩ C contains those elements in all three of A, B, and C. Thus,

A ∩ B ∩ C ={0}.

Computer Representation of Sets

Assume that the universal set U is finite first, specify an arbitrary ordering of the elements of U, for instance $a_1, a_2, \ldots, a_n$. Represent a subset A of U with the bit string of length n, where the ith bit in this string is 1 if a_i belongs to A and is 0 if a_i does not belong to A

EXAMPLE

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and the ordering of elements of U has the elements in increasing order; that is, $a_i = i$. What bit strings represent the subset of all odd integers in U, the subset of all even integers in U, and the subset of integers not exceeding 5 in U?

- The bit string that represents the set of odd integers in U, namely, A={1, 3, 5, 7, 9}, 10 1010 1010.

Similarly,

- we represent the subset of all even integers in U, namely, B={2, 4, 6, 8, 10}, by the string 01 0101 0101.

- The set of all integers in U that do not exceed 5, namely, C={1, 2, 3, 4, 5}, is represented by the string 11 1110 0000.

What is the bit string for the complement of A

the complement of this set is obtained by replacing 0s with 1s and vice versa. This yields the string 01 0101 0101, which corresponds to {2, 4, 6, 8, 10}

- The bit string for the union of A, B

11 1110 0000 ∨ 10 1010 1010 = 11 1110 1010
which corresponds to the set {1, 2, 3, 4, 5, 7, 9}.

- The bit string for the intersection of these sets is

11 1110 0000 ∧ 10 1010 1010 = 10 1010 0000,
which corresponds to the set {1, 3, 5}.

Lecture 4


Thank
You!